JAN REIMANN, PENN STATE MATH, UNIVERSITY PARK

PATTERNS

Alluvial Fan, China (Photo: NASA)

Himmalayas (Photo: NASA)

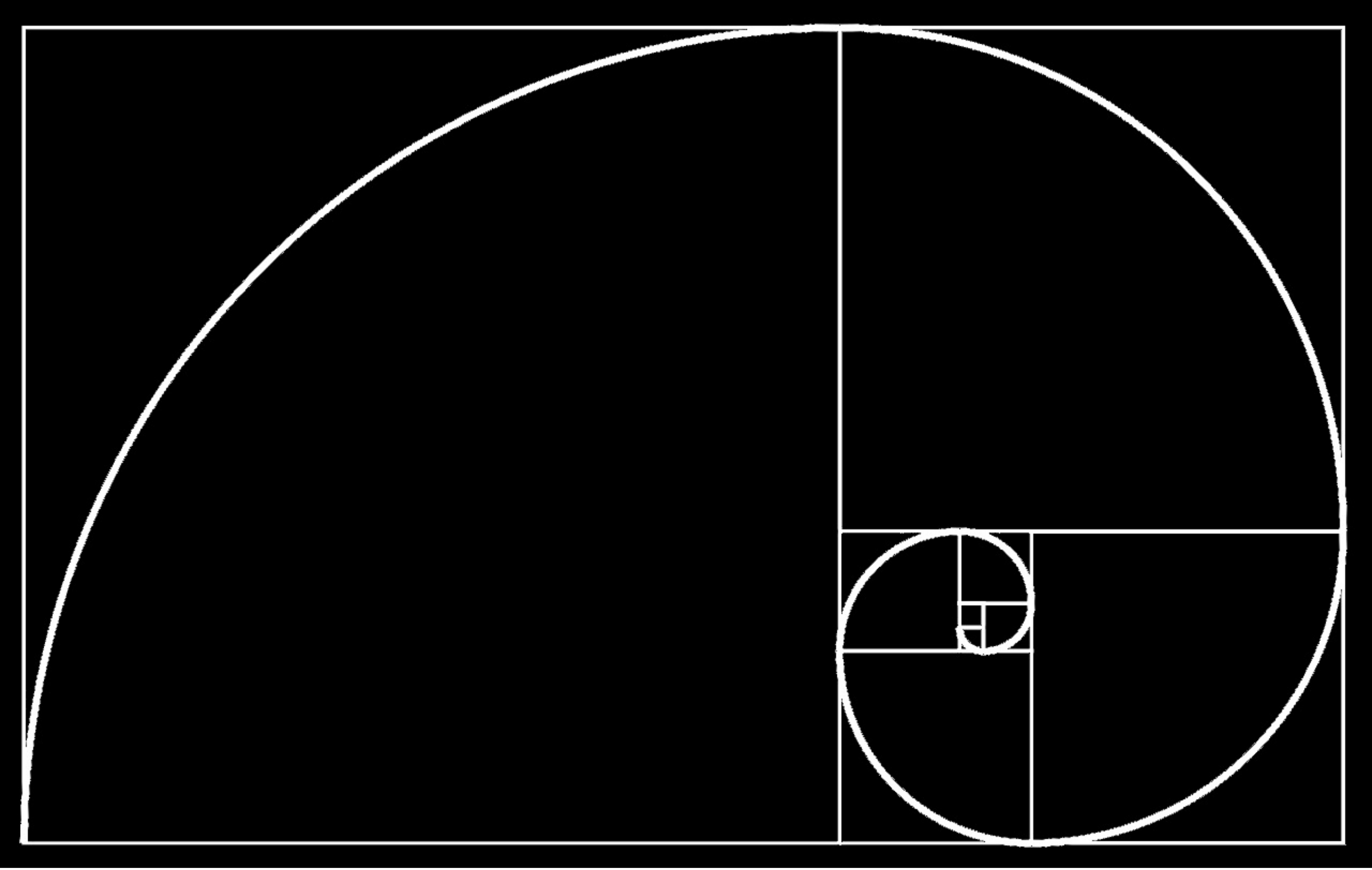
Patterns exist only where we know how to see them

Curiosity (and scientific endeavor) teaches us to see them

Himmalayas (Photo: NASA)

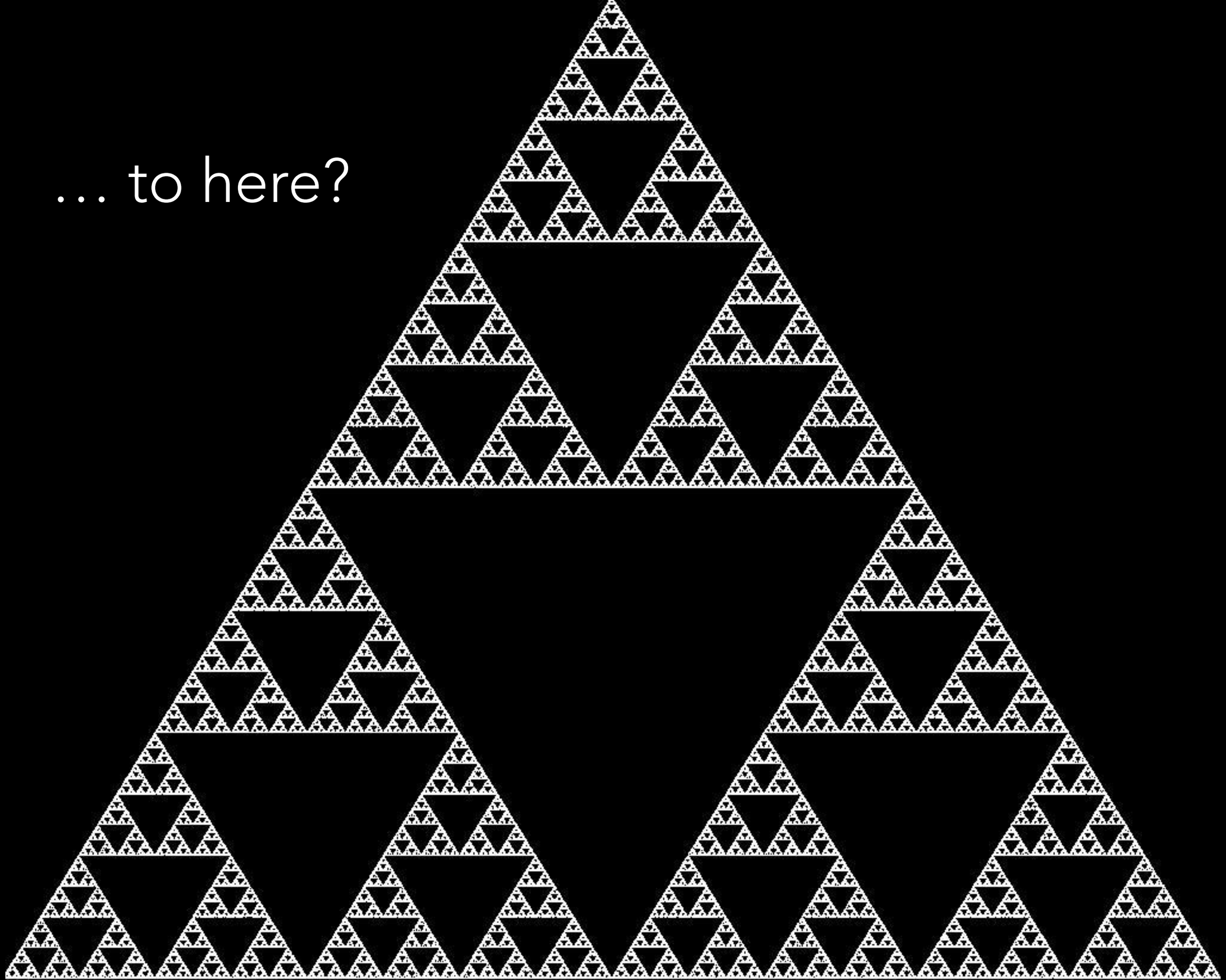
"Aloe polyphylla 1" by Stan Shebs

How do you get from here…




And from here…

… to here?



3000 years of mathematics…

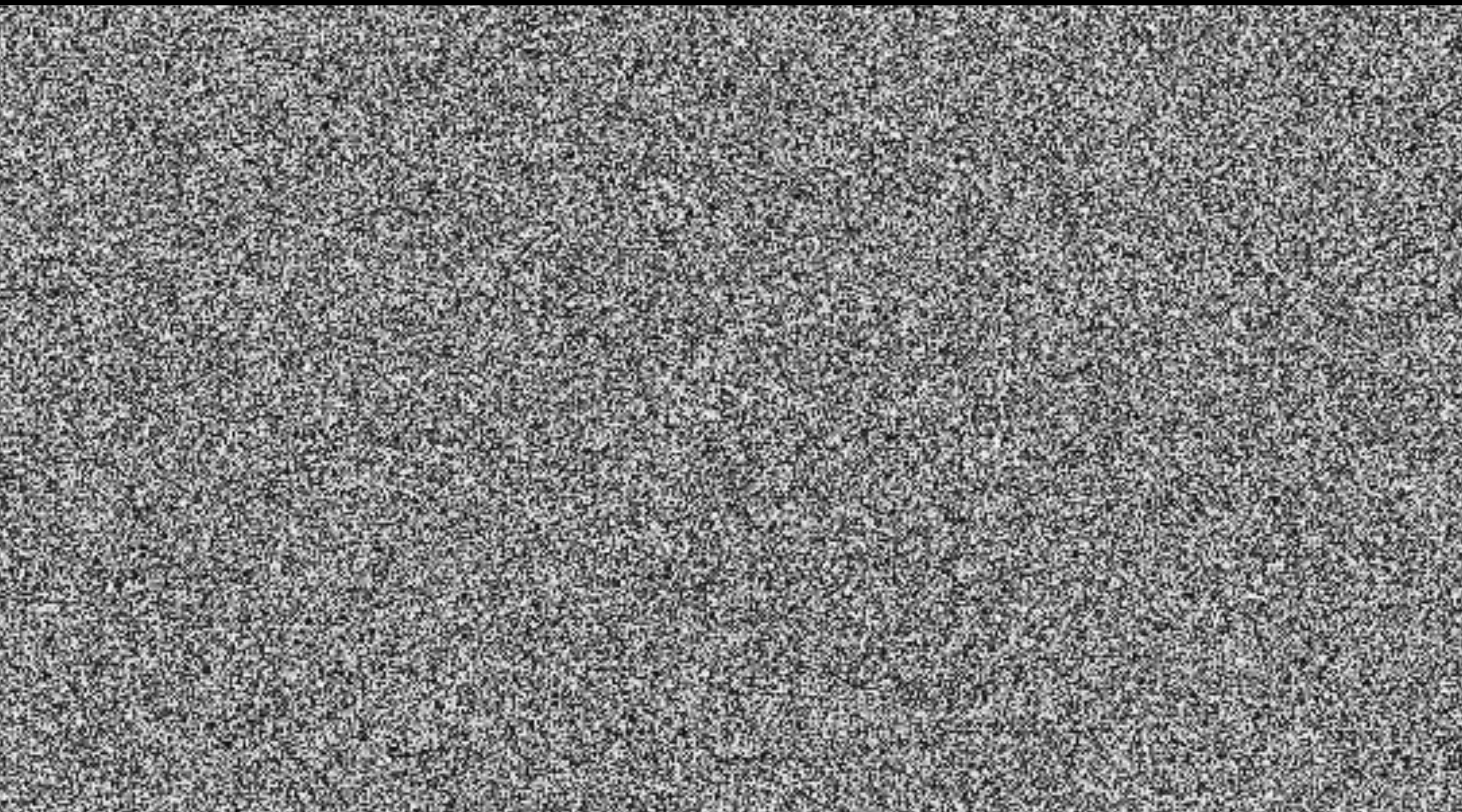
… and the human struggle with it

CHAOS

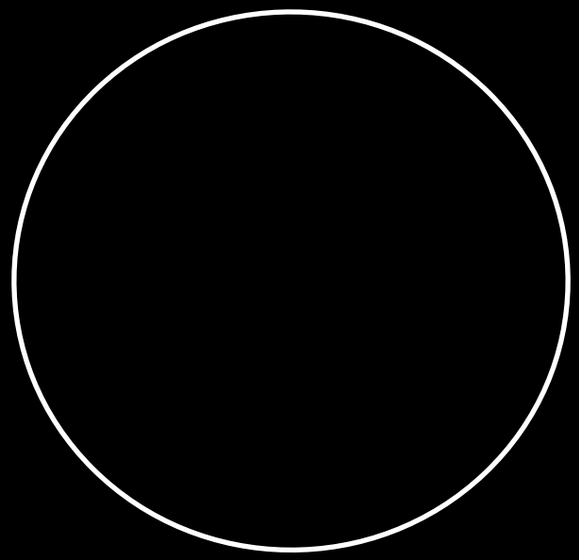
Ovid, *Metamorphoses* (8 AD):

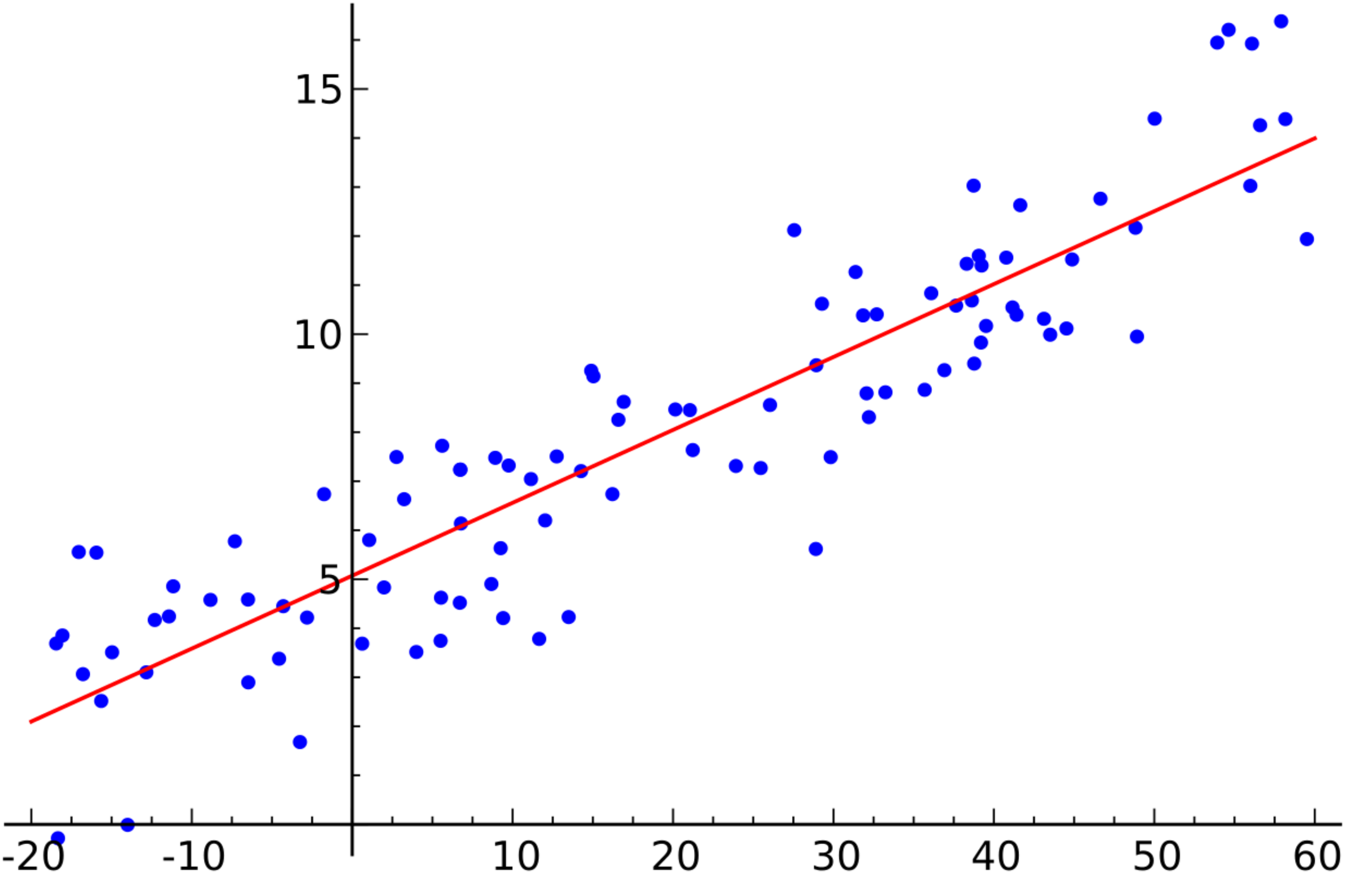
"Before there was earth or sea
or the sky that covers
everything, Nature appeared
the same throughout the
whole world: what we call
chaos: a raw confused mass,
nothing but inert matter, badly
combined discordant atoms of
things, confused in the one
place."

Today we would probably call that
random noise



The fundamental patterns





Linear regression

But why do we sometimes find the occurrence of simple patterns in nature surprising?

San Andreas Fault at Carrizo Plain
(by John Wiley, from Wikimedia Commons)

S P Crater, AZ

(Photo: NASA)

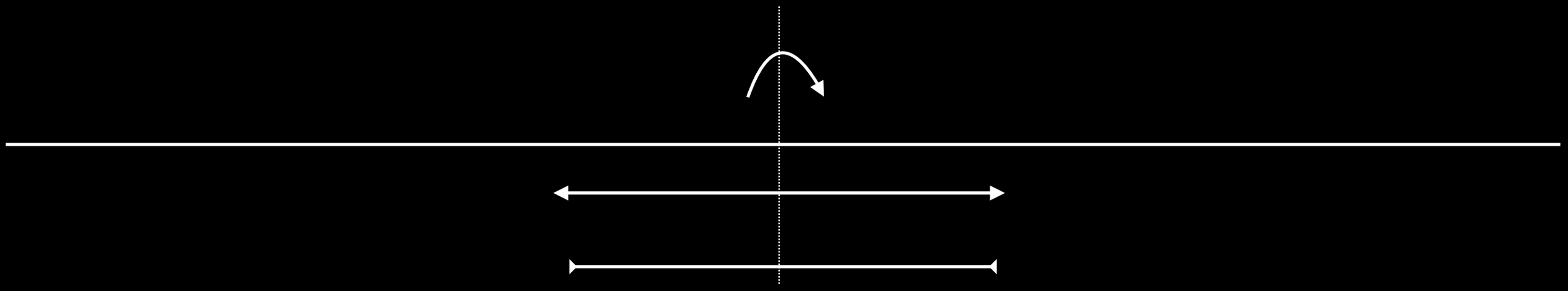
THE ANCIENT WORLD

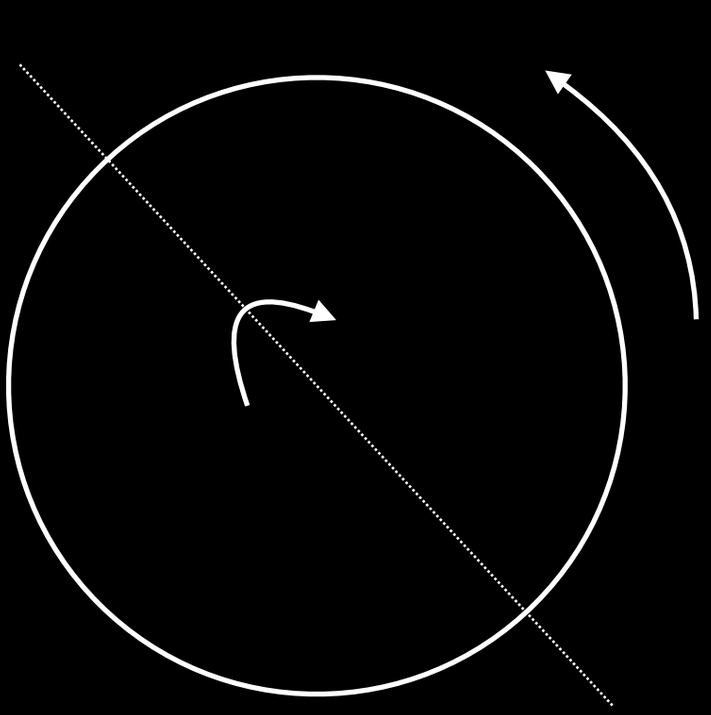
Greek architecture = a lot of fundamental patterns

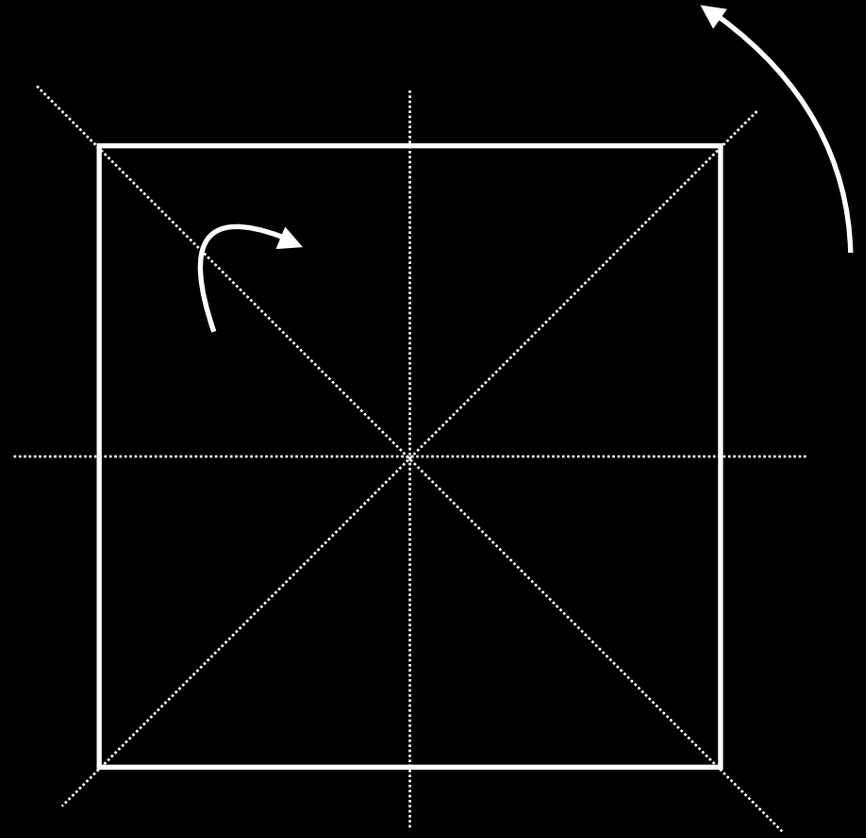
(IS THERE A PATTERN OF DECAY?)

Greek architecture, Tennessee style

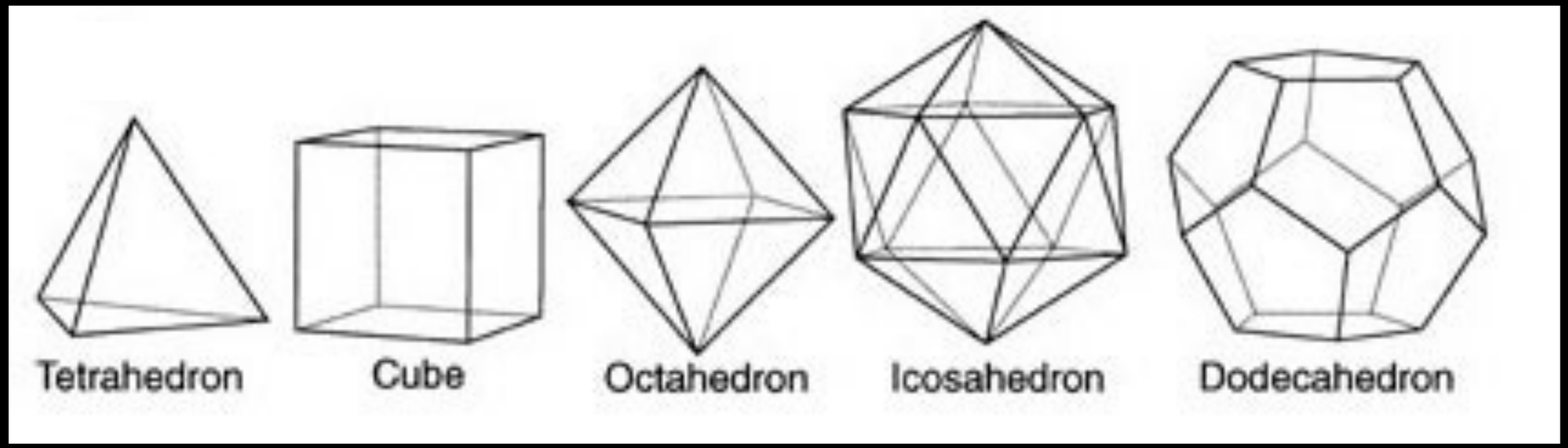
Fundamental pattern: Symmetry



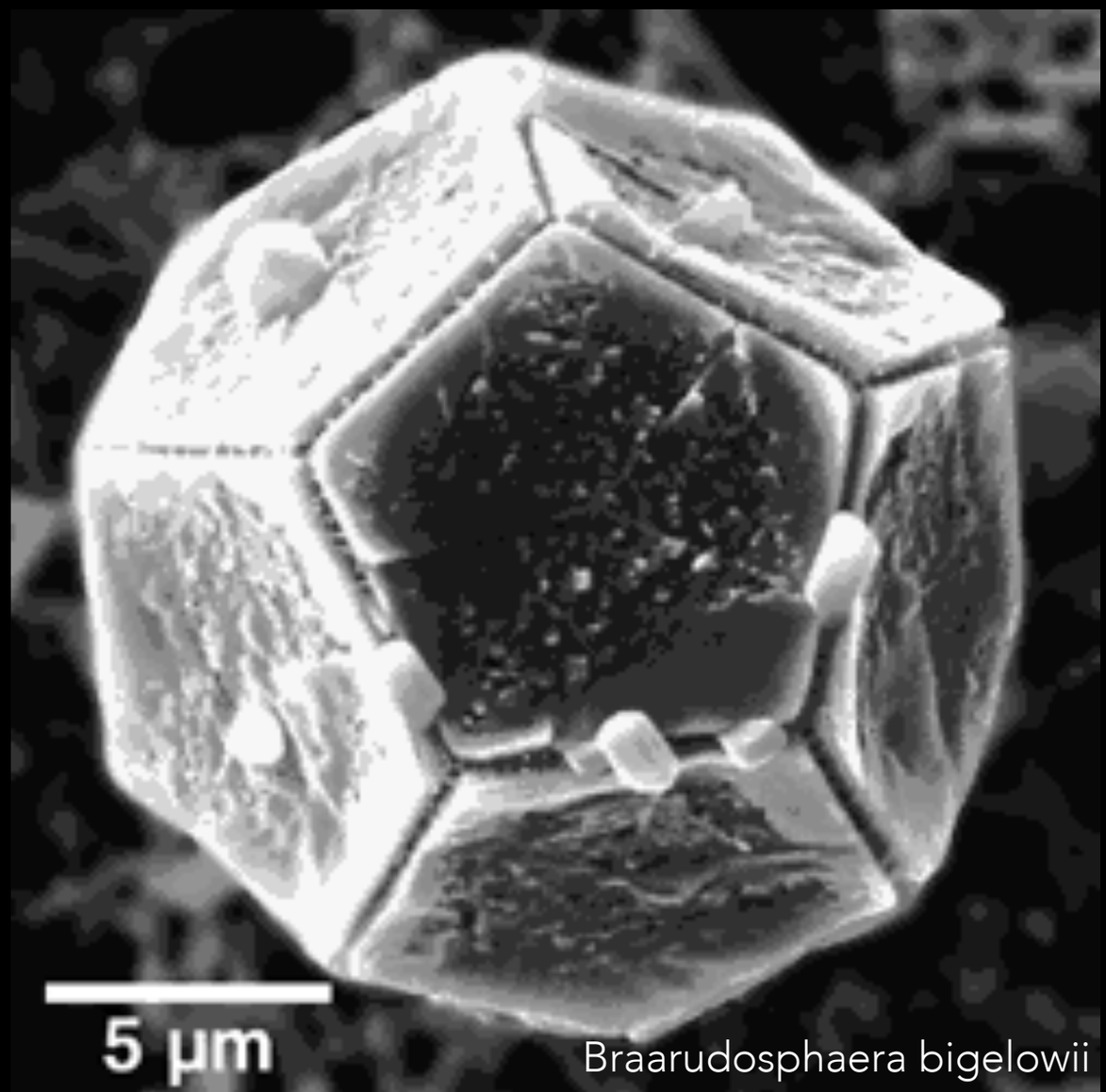




The Platonic Solids

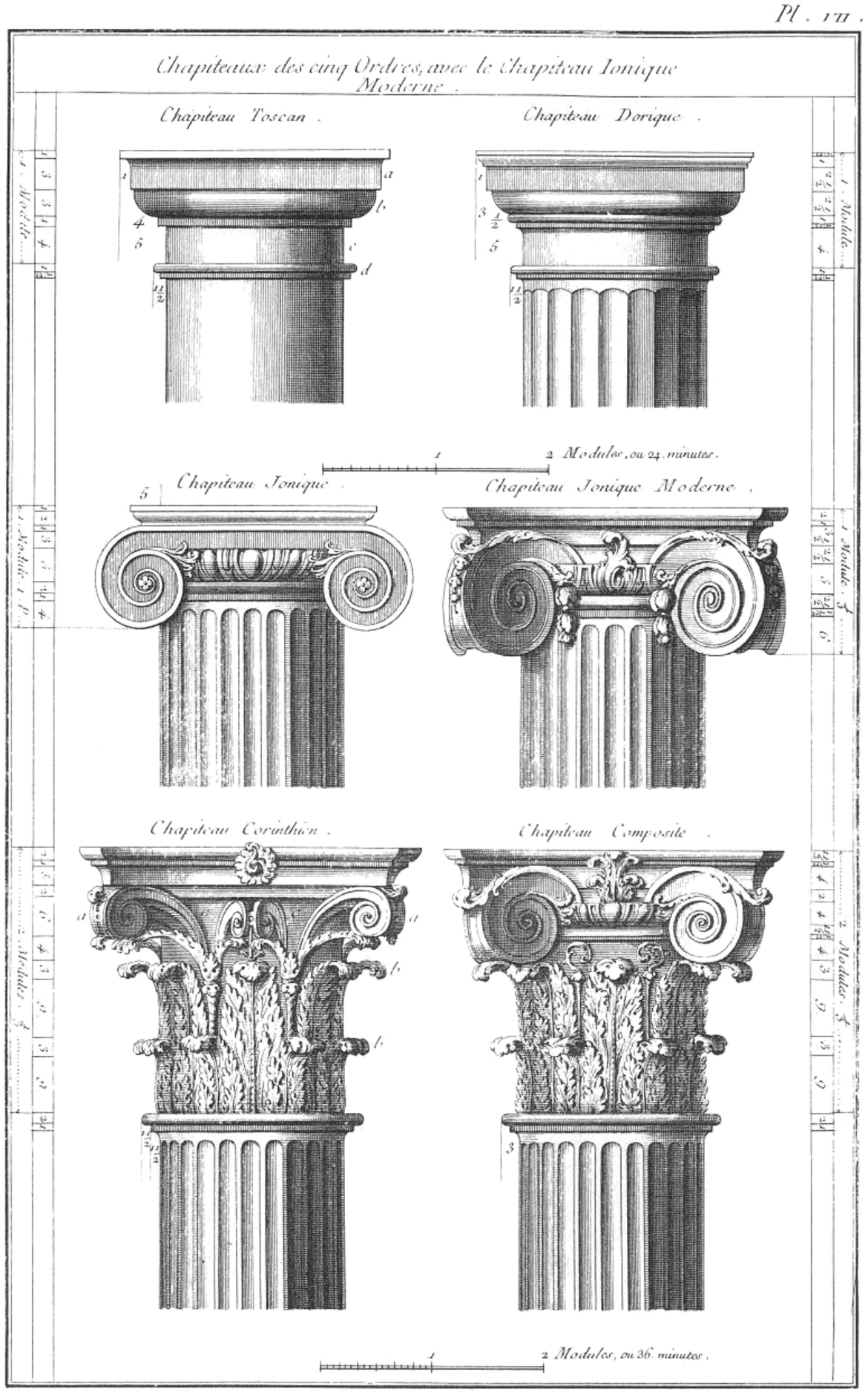


Pyrit (Eisensulfid), Navajun/Spanien, Foto und Copyright: T. Seilnacht



Braarudosphaera bigelowii

But the patterns evolved over time…



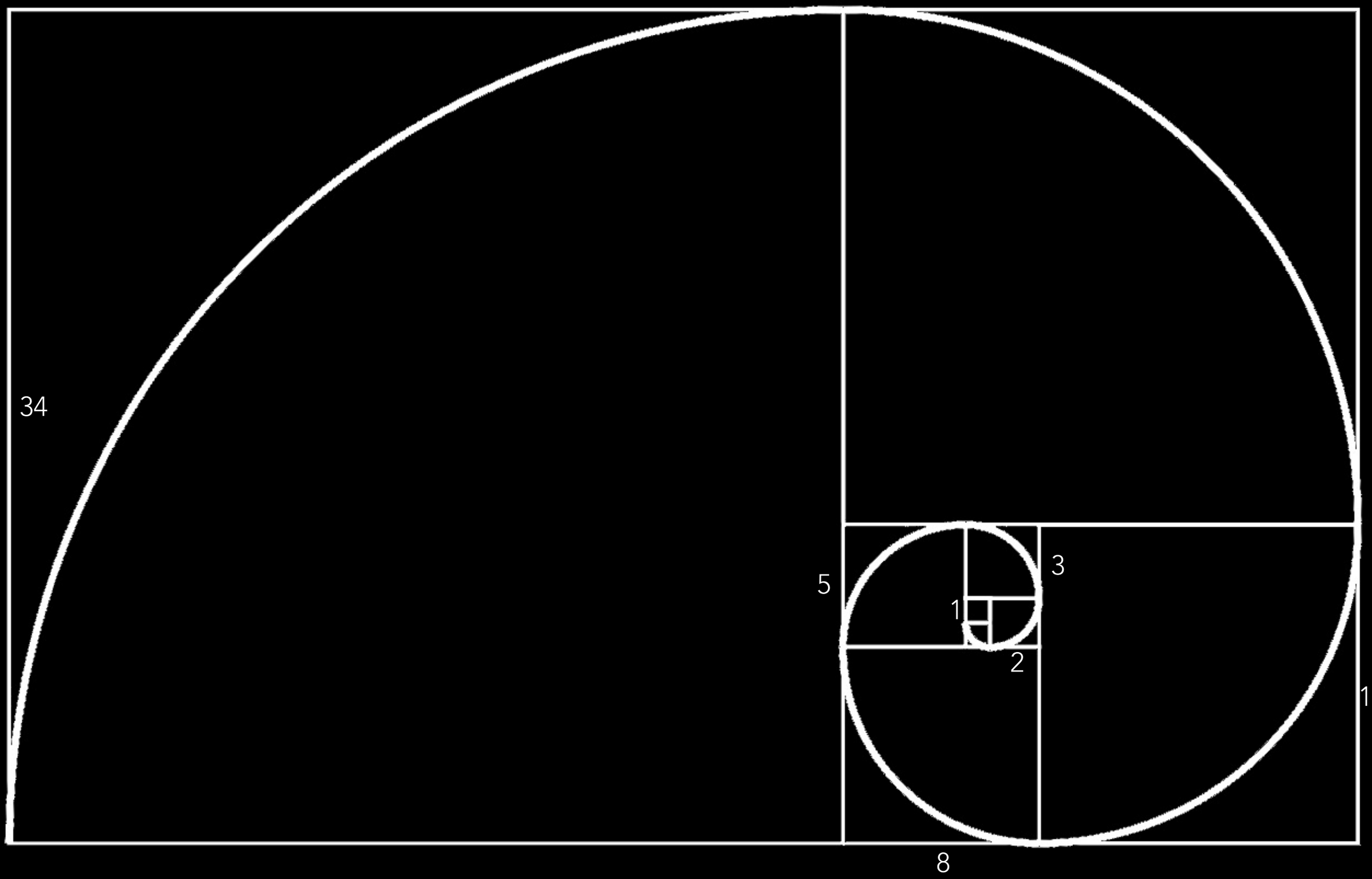
Architecture

Digression: Spirals

Messier 51 “Whirpool” Galaxy (Photo: NASA)

"Aloe polyphylla 1" by Stan Shebs

"Nautilus Cutaway with Logarithmic Spiral" by Dicklyon - (via Wikimedia)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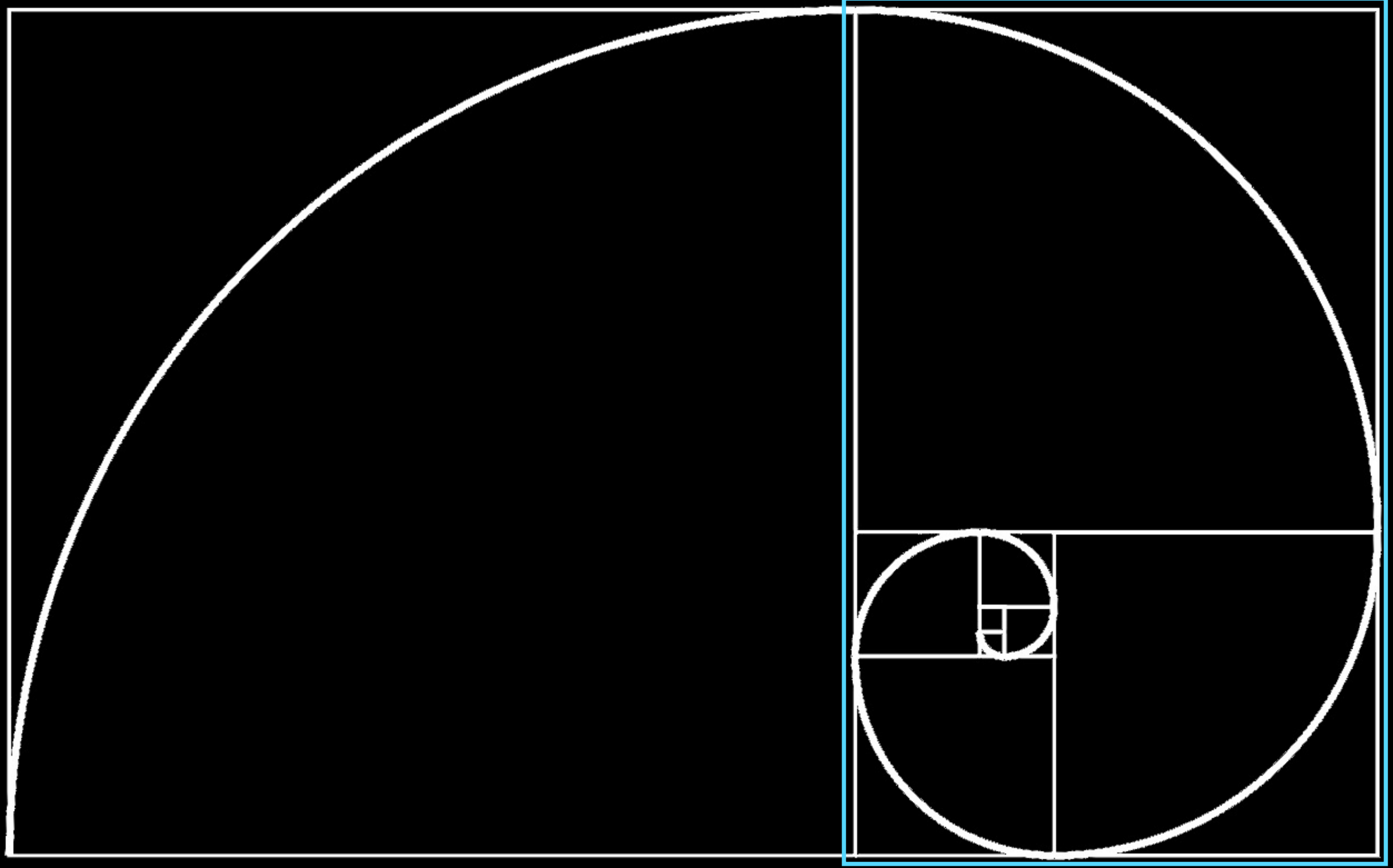
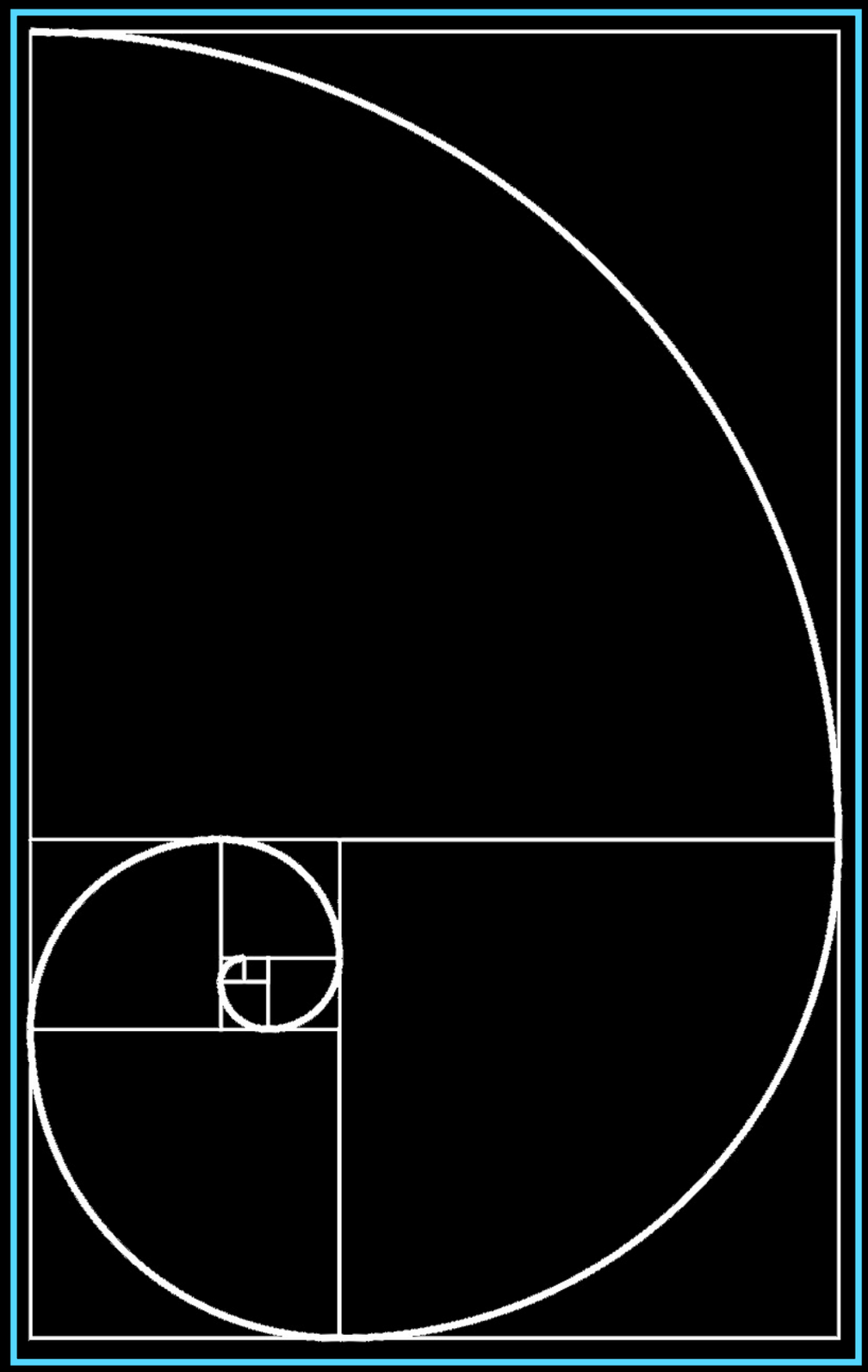
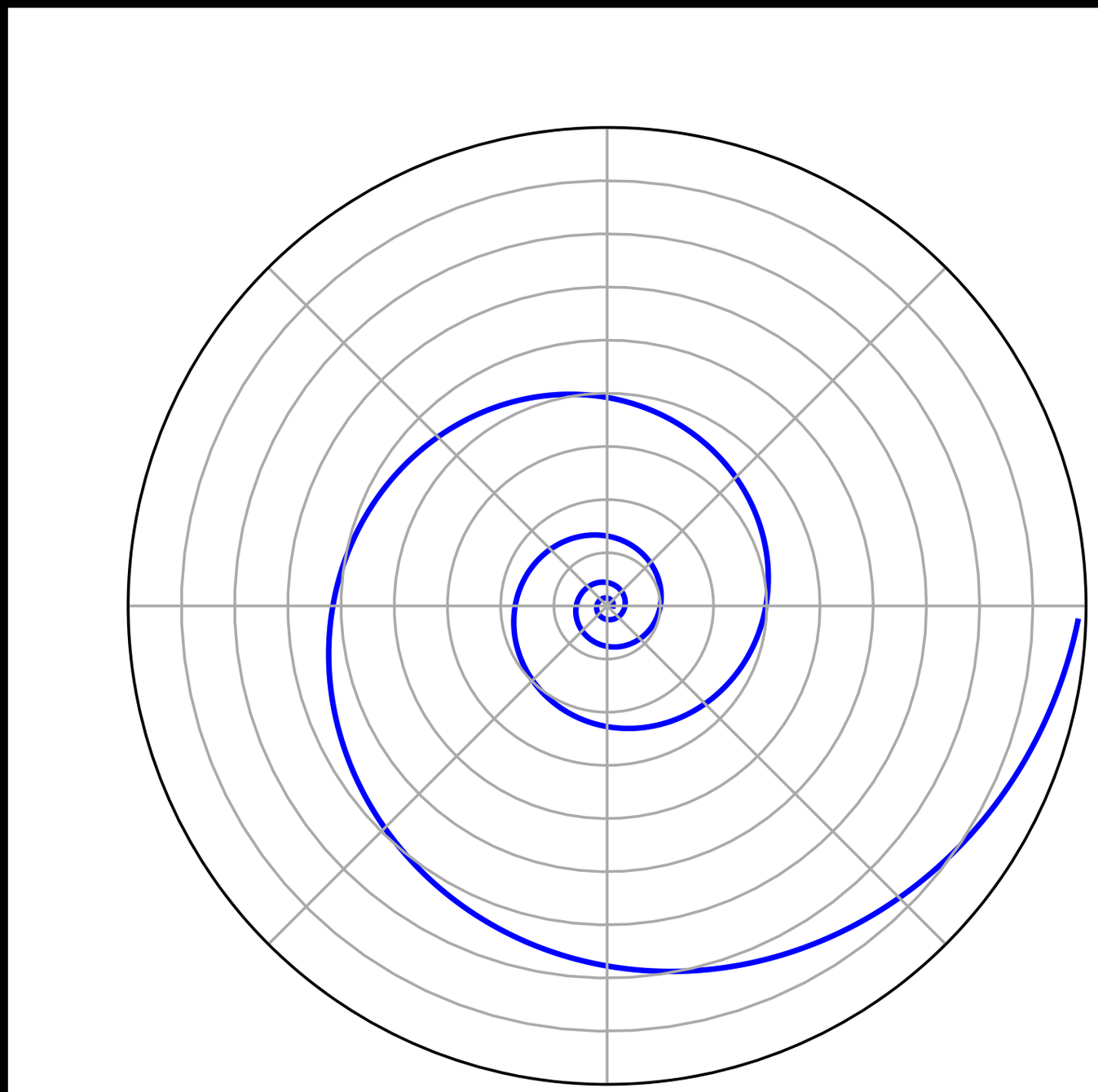




Logarithmic Spirals
a synthesis of circle and line

Symmetry:
Rotation + Scaling



THE AGE OF BAROQUE

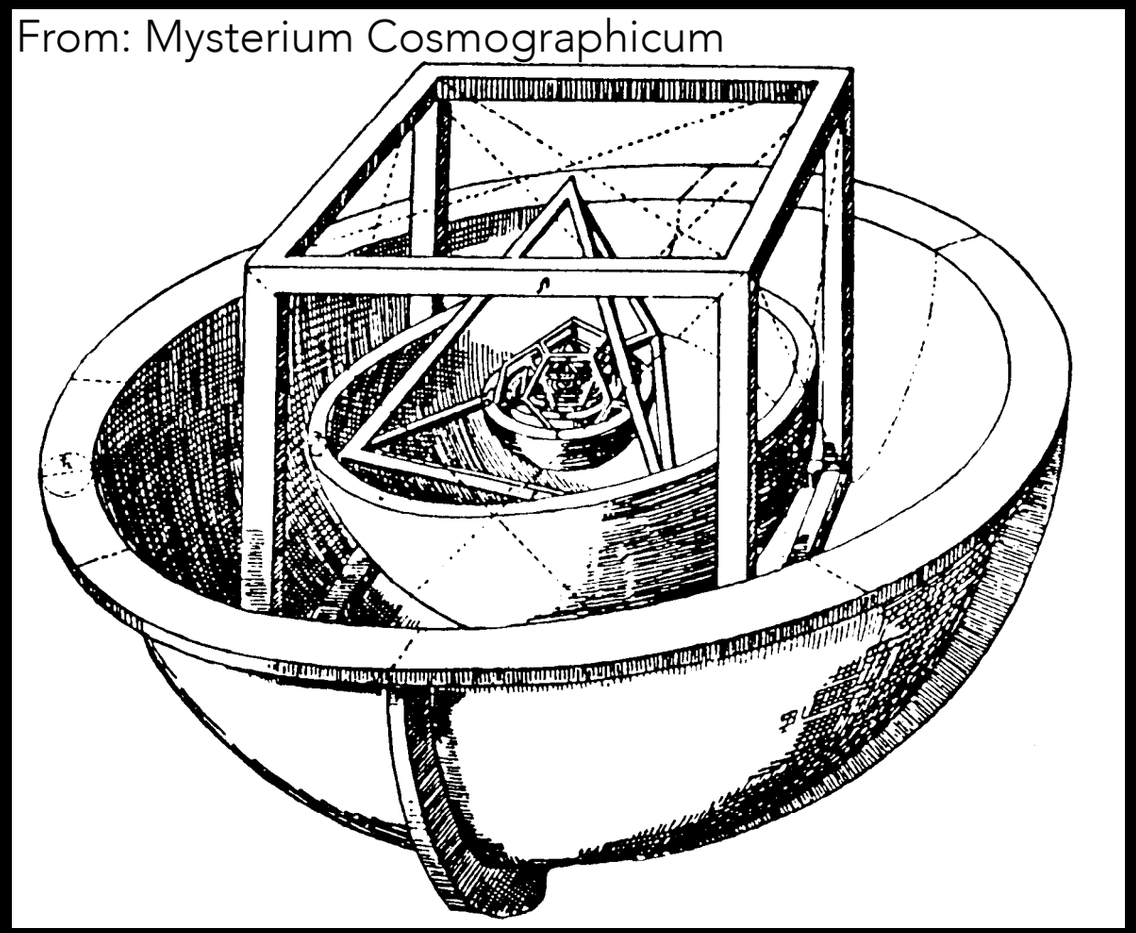
DAWN OF THE INFINITESIMAL

Kepler – from circle to ellipsis

1st law: The orbit of a planet is an ellipse with the Sun at one of the two foci.

But Kepler also believed in a strong, pre-established harmony of the universe, based on fundamental geometric patterns.

From: Mysterium Cosmographicum



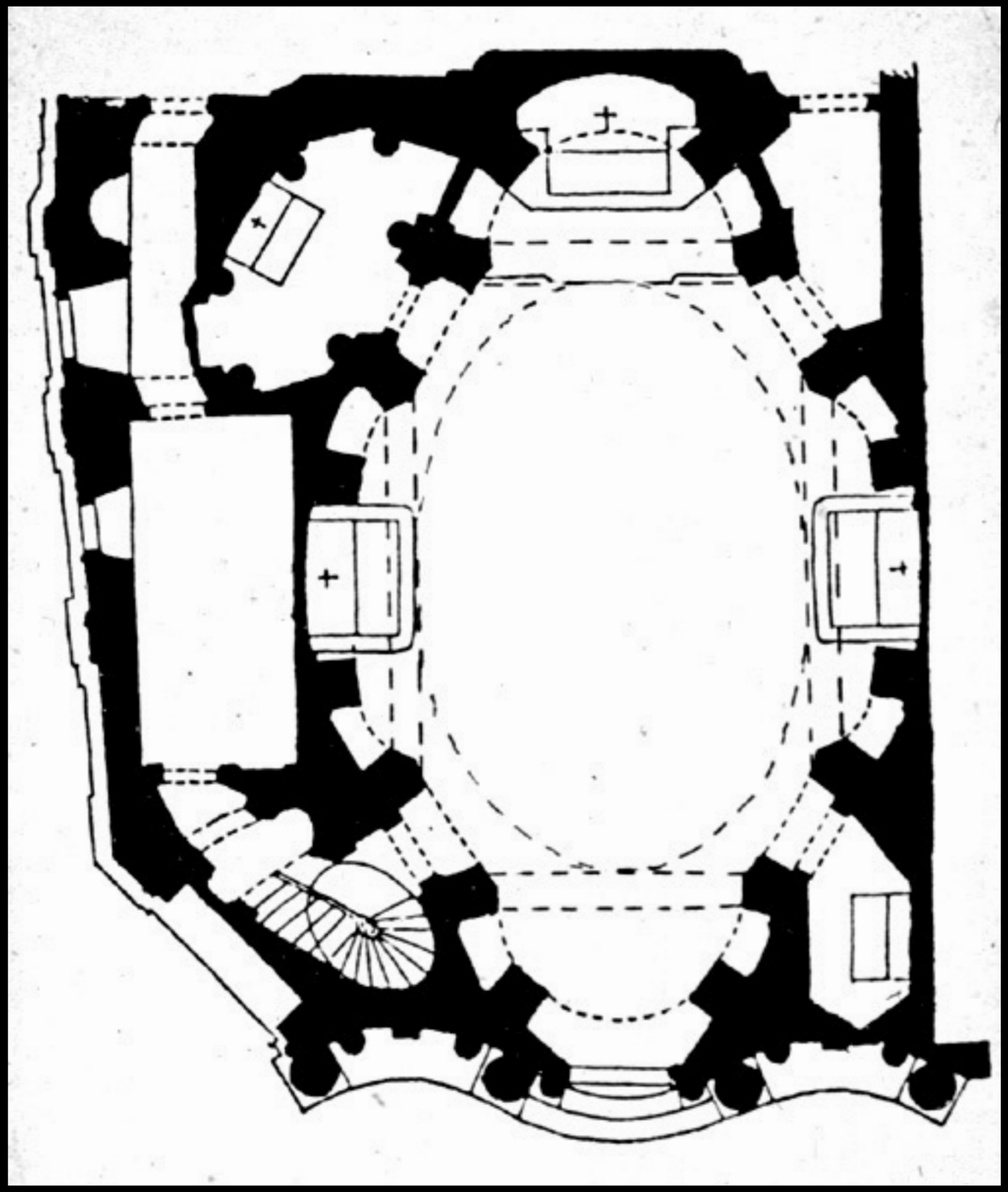
Rome – Bernini vs Borromini

Francesco Borromini

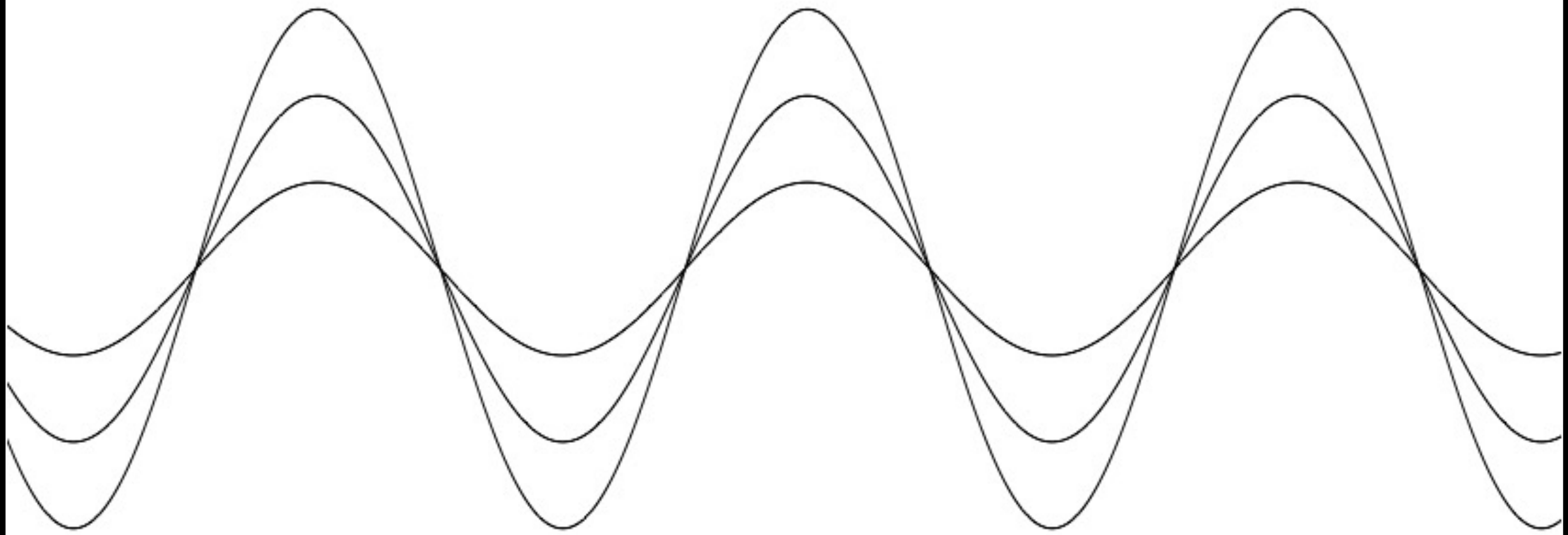
Gianlorenzo Bernini

San Carlo alle Quattro Fontane

"SCarloQuattroFontaneRome2", Welleschik (Wikimedia Commons)




SANCTISS TRINITATI BEATOQ CAROLO BORROM
MDCXL



Newton and Leibniz:
Triumph of the Infinitesimal

Gottfried Wilhelm Leibniz

Isaac Newton

In a smooth world, everything looks locally like a line.

19TH CENTURY

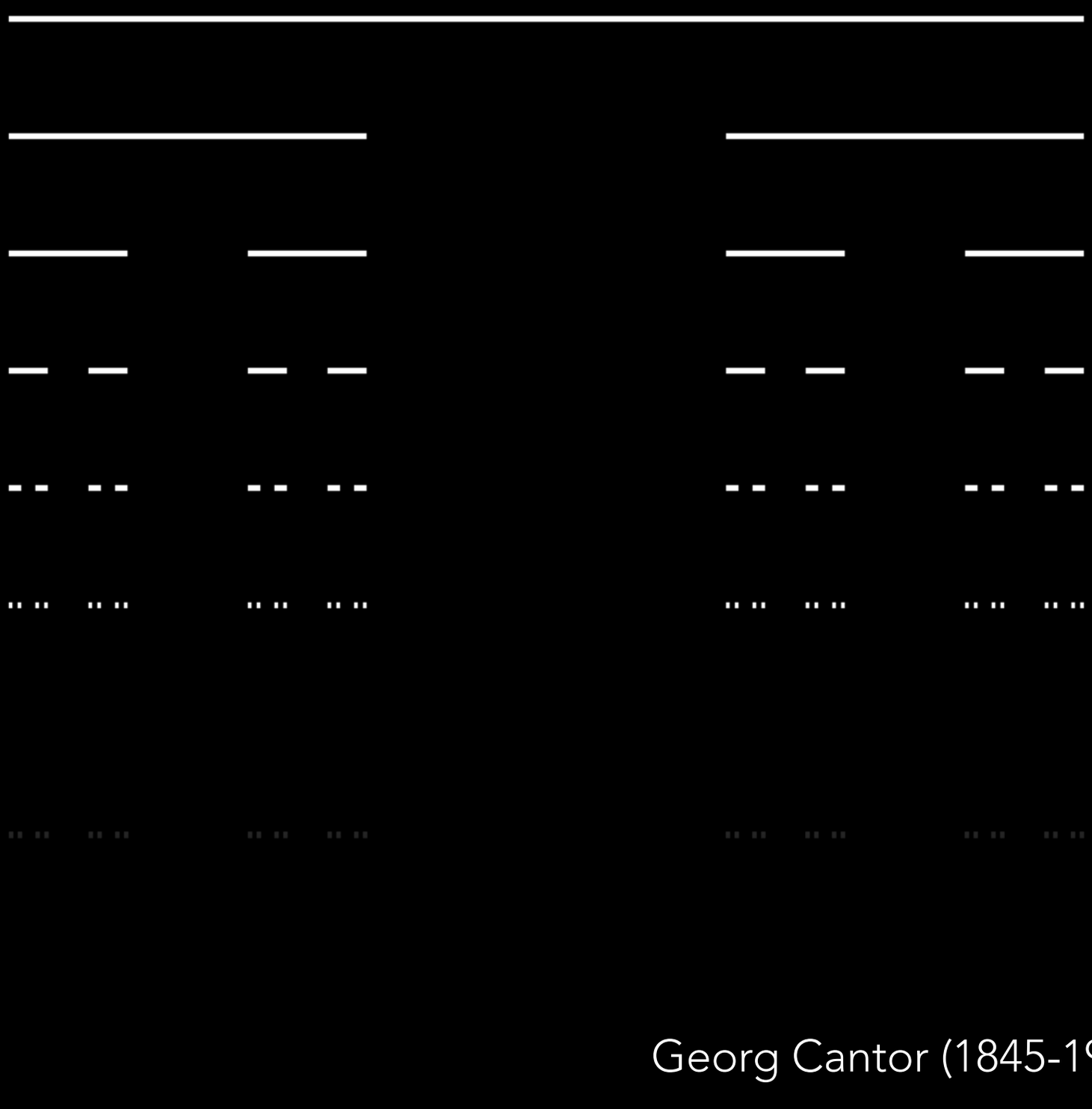
INFINITESIMAL ABNORMALITIES

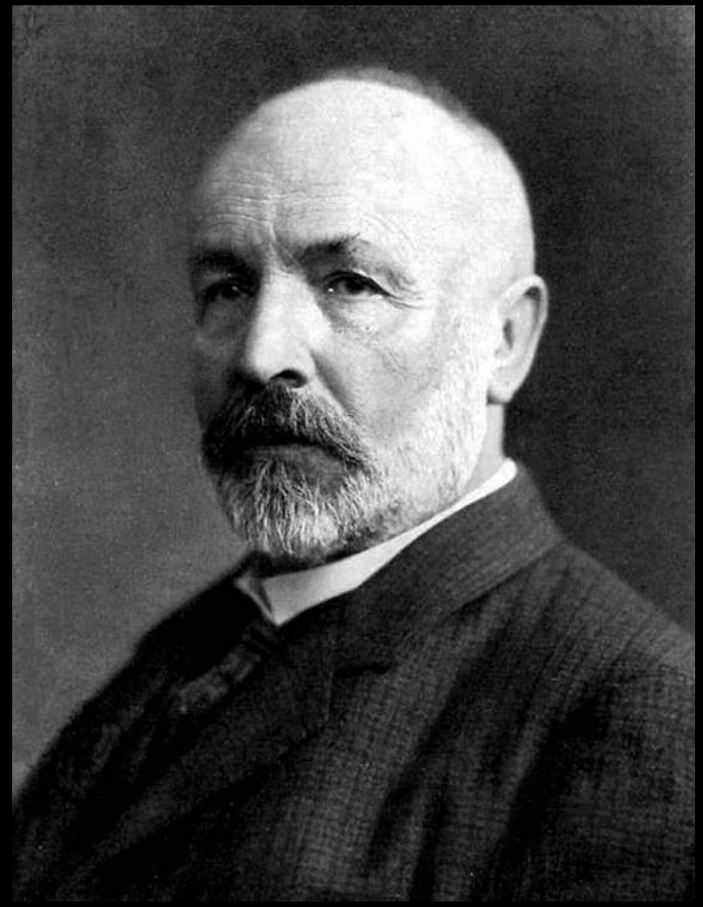
This does not look smooth.

This does not look smooth.

In fact, in many cases we find
smooth patterns rather surprising…

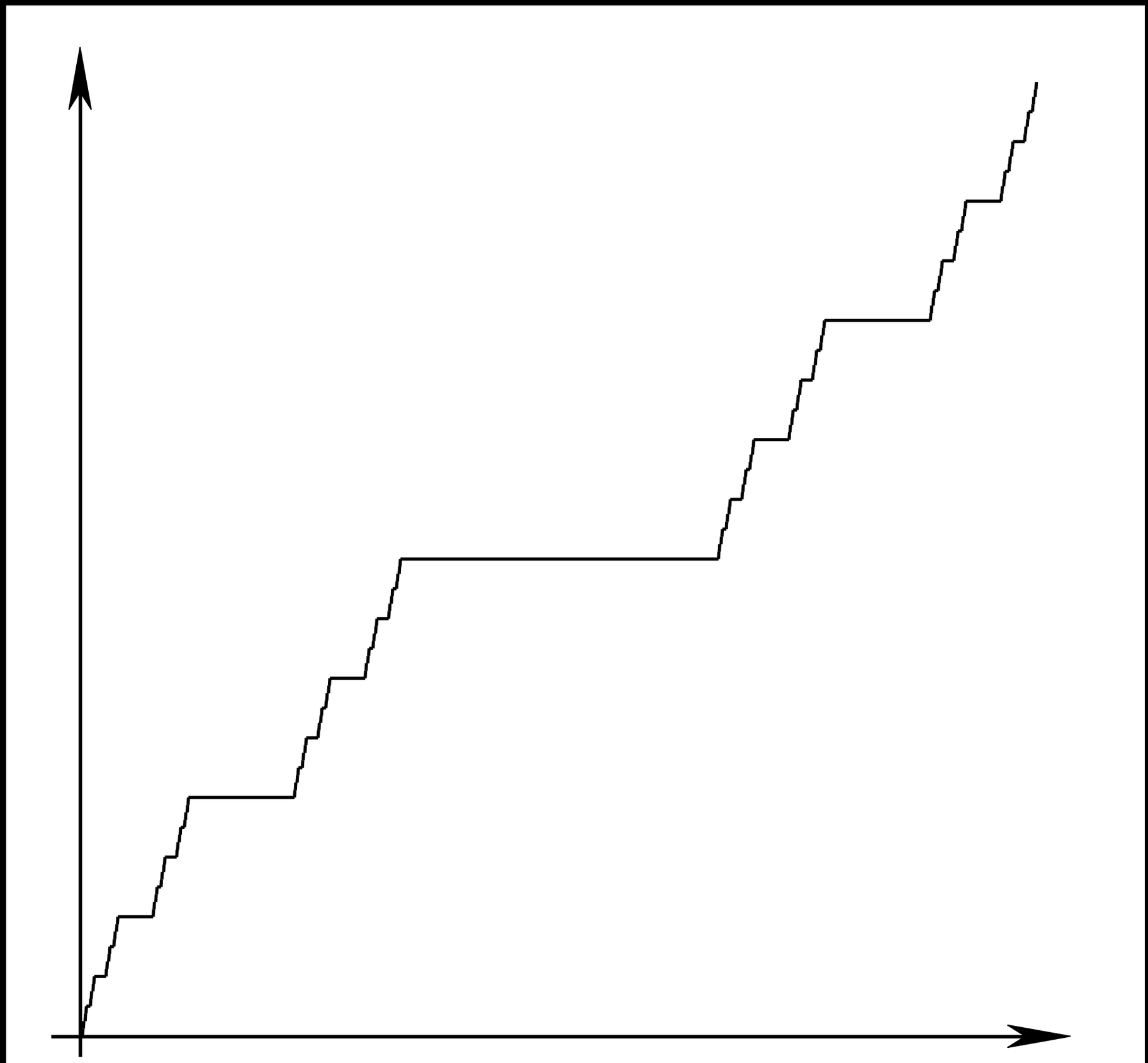
The Cantor Set



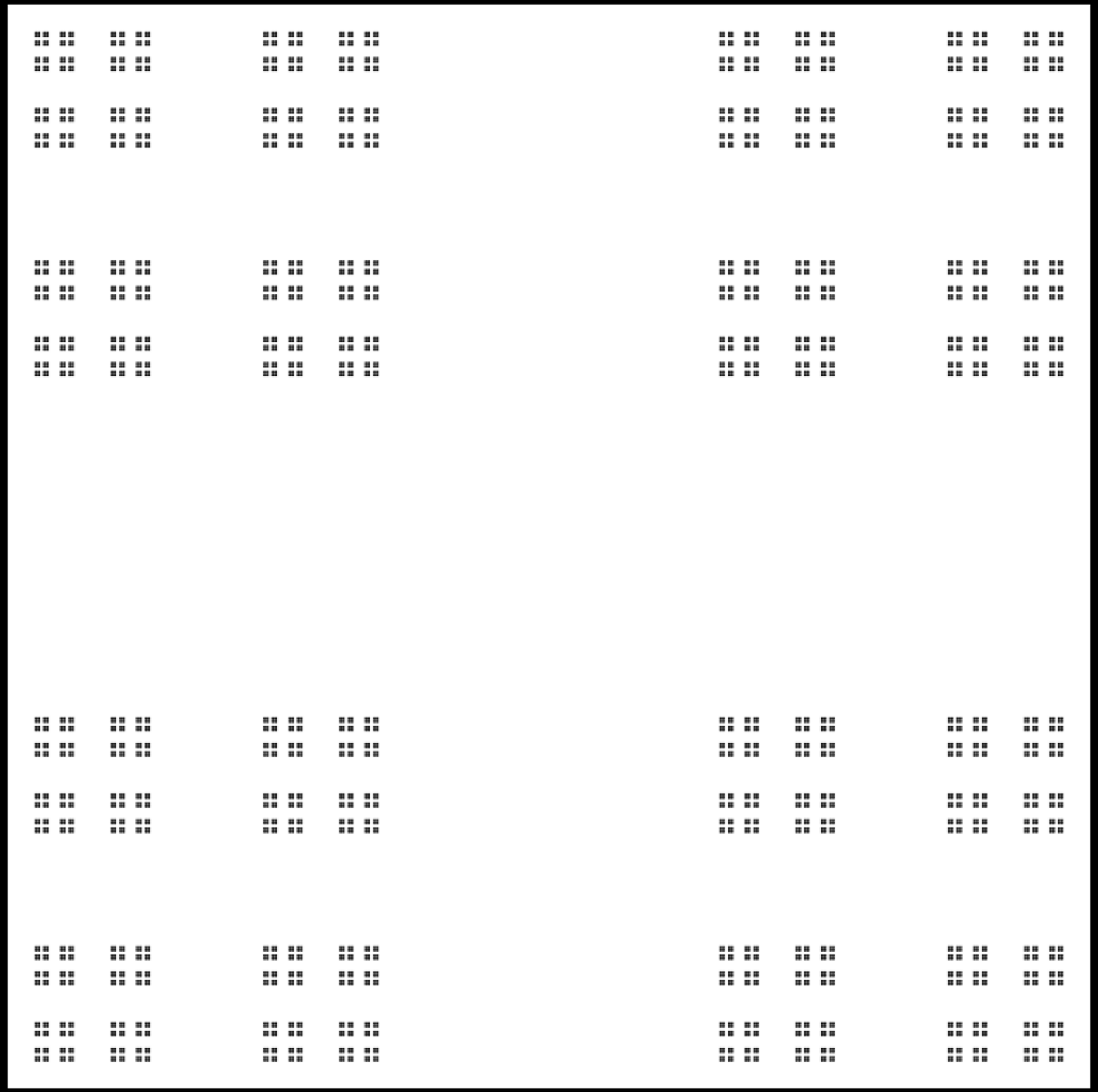


Georg Cantor (1845-1918)

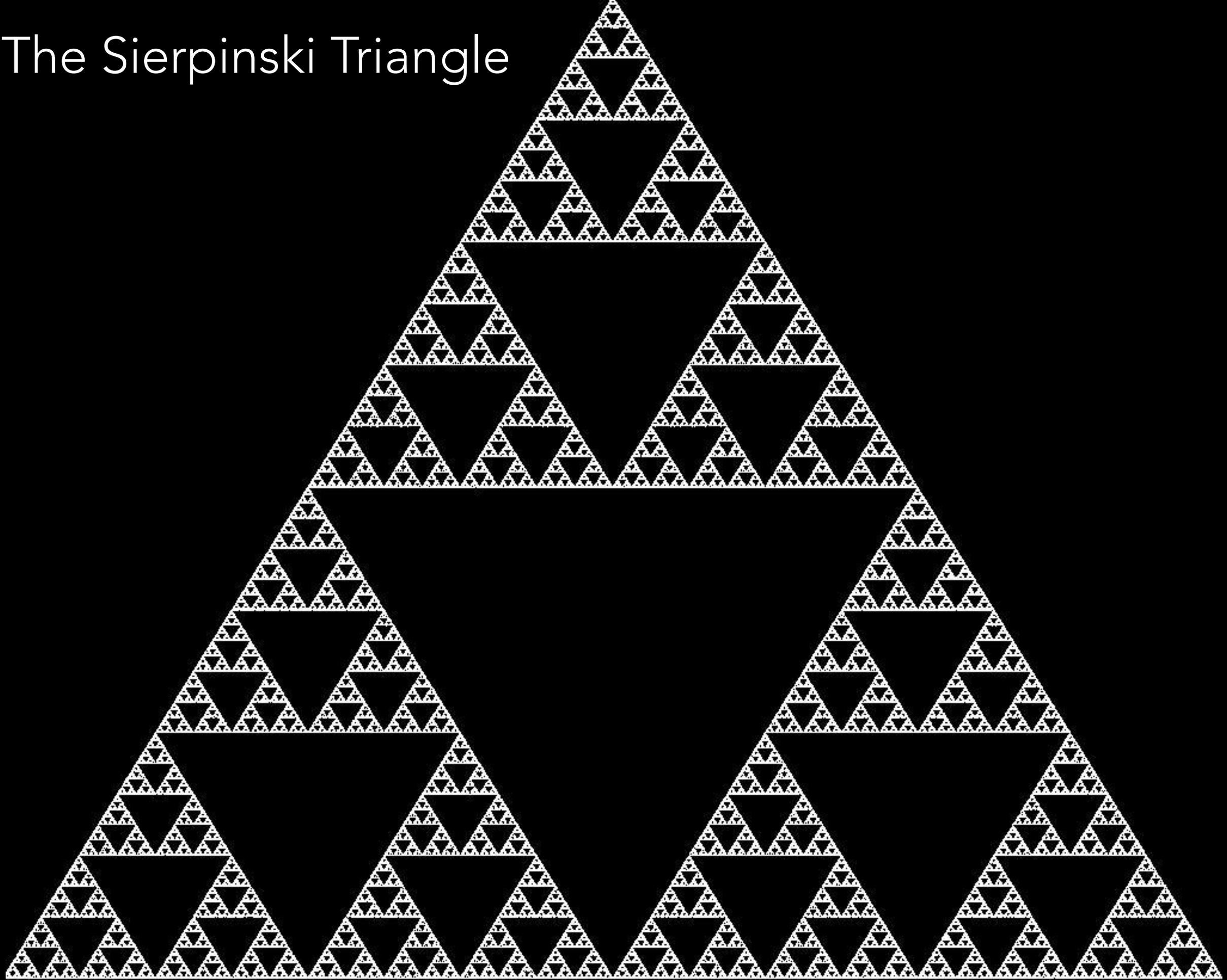
Not smooth at all…



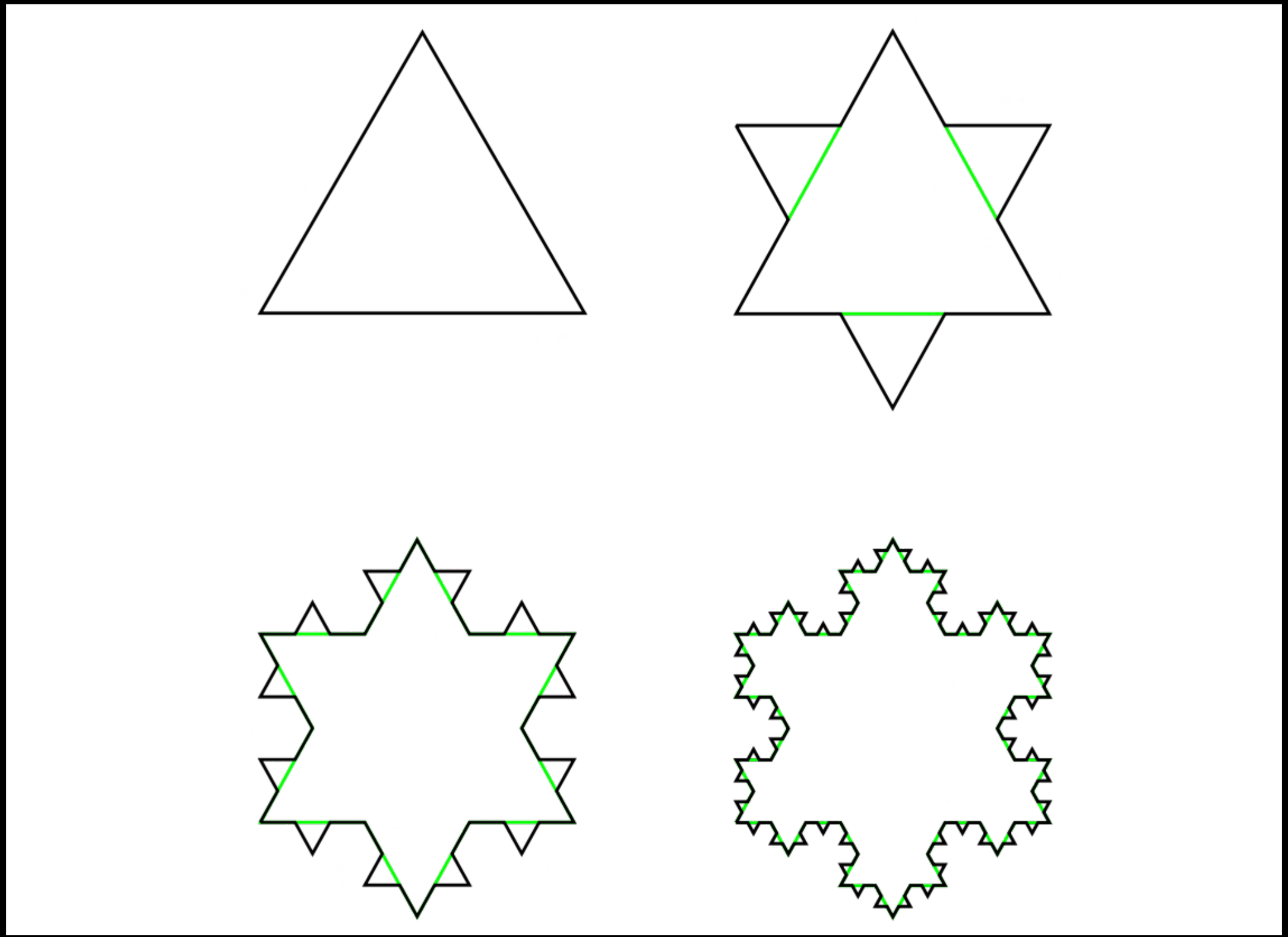
The Cantor Dust



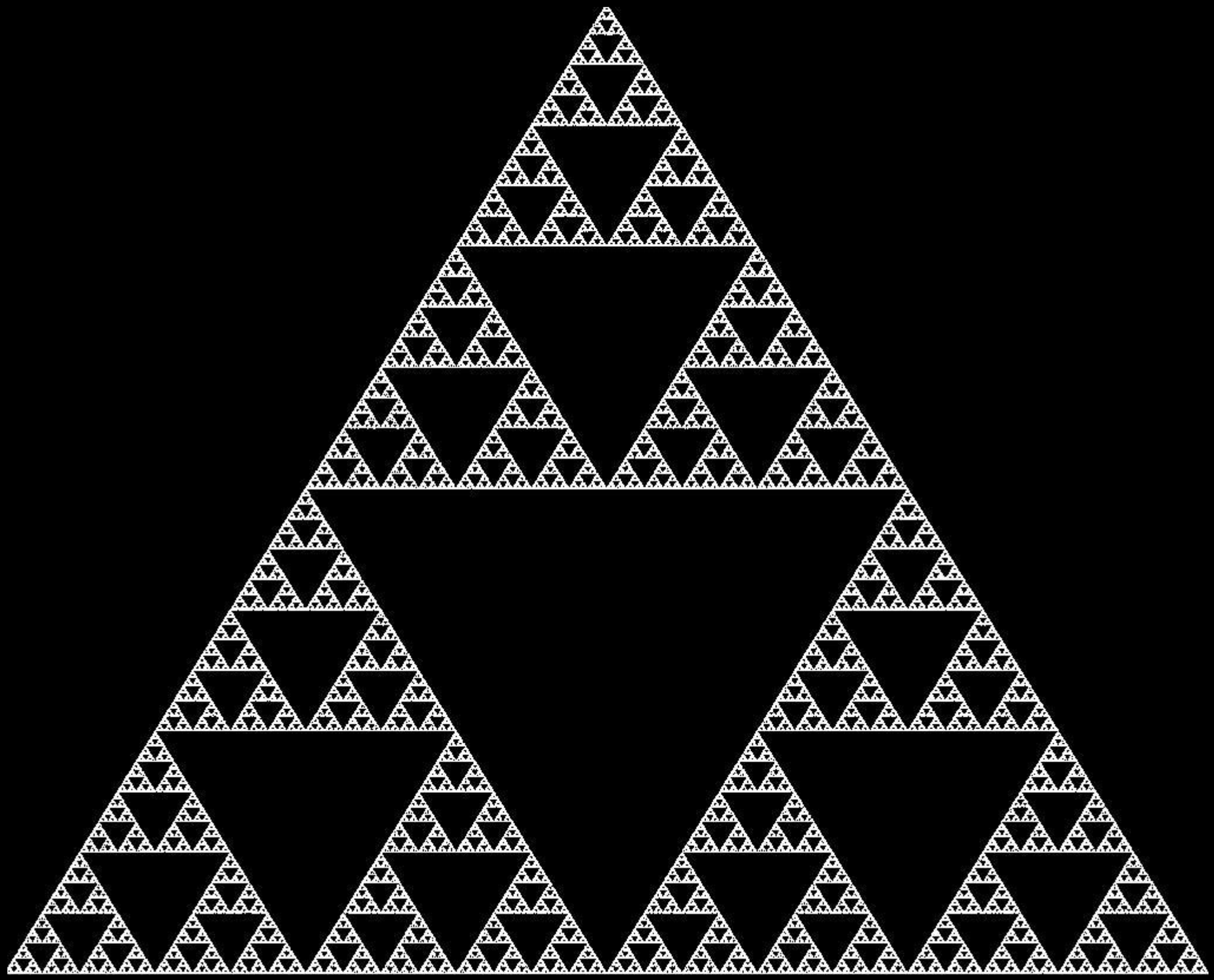
The Sierpinski Triangle



The Koch Snowflake



What happens if we “zoom in” any of these sets?



We obtain the original set back!

This behavior is called “self-similarity”.
It is a different kind of symmetry.

Remember the spirals.

Important difference: These sets are similar
to multiple parts of themselves.

20TH CENTURY

THE DAWN OF FRACTALS

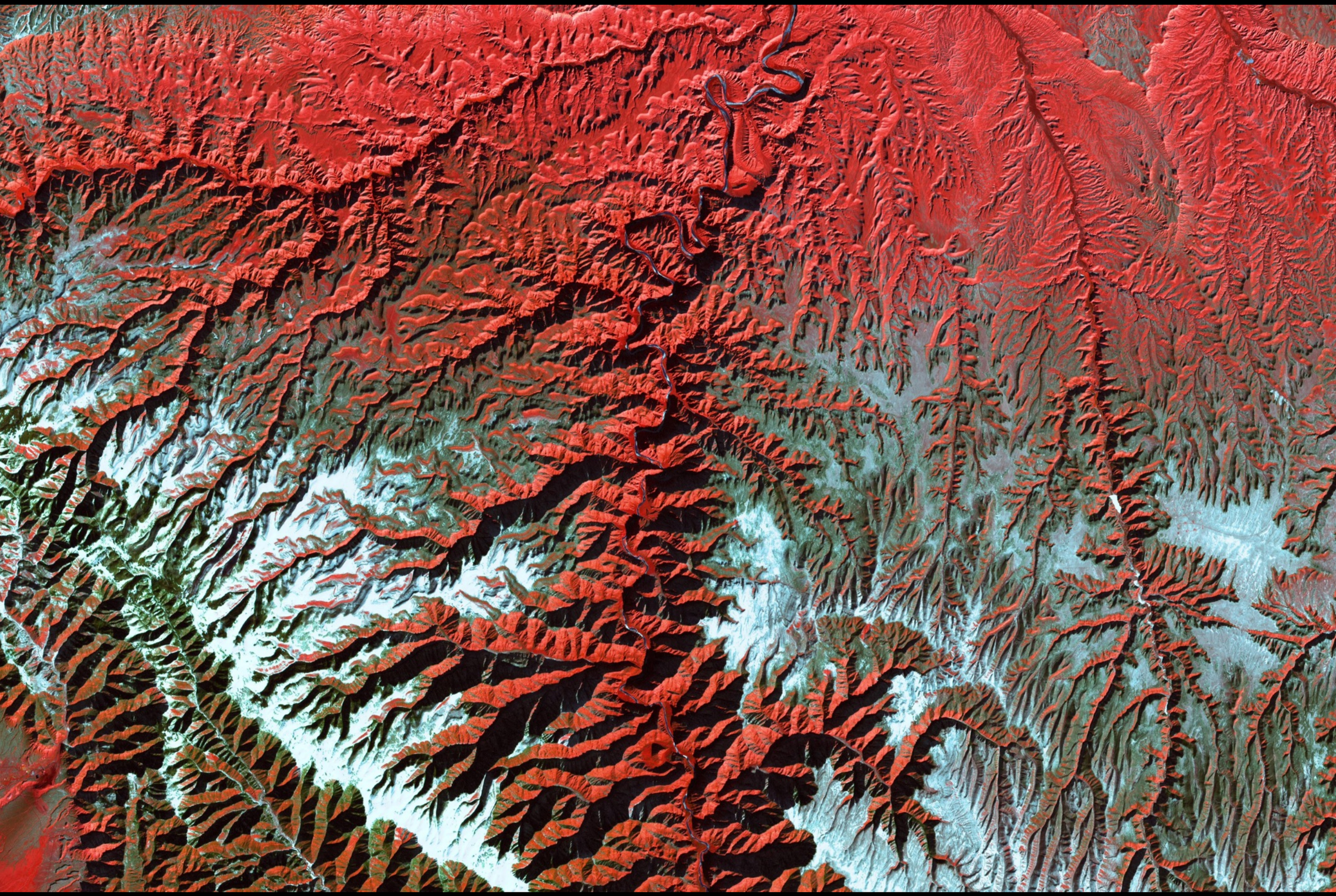
What do these strange mathematical sets have to do with patterns in nature?

There is plenty of self-similarity…

"Sa-fern" by Sanjay ach (via Wikimedia)

There is plenty of self-similarity…

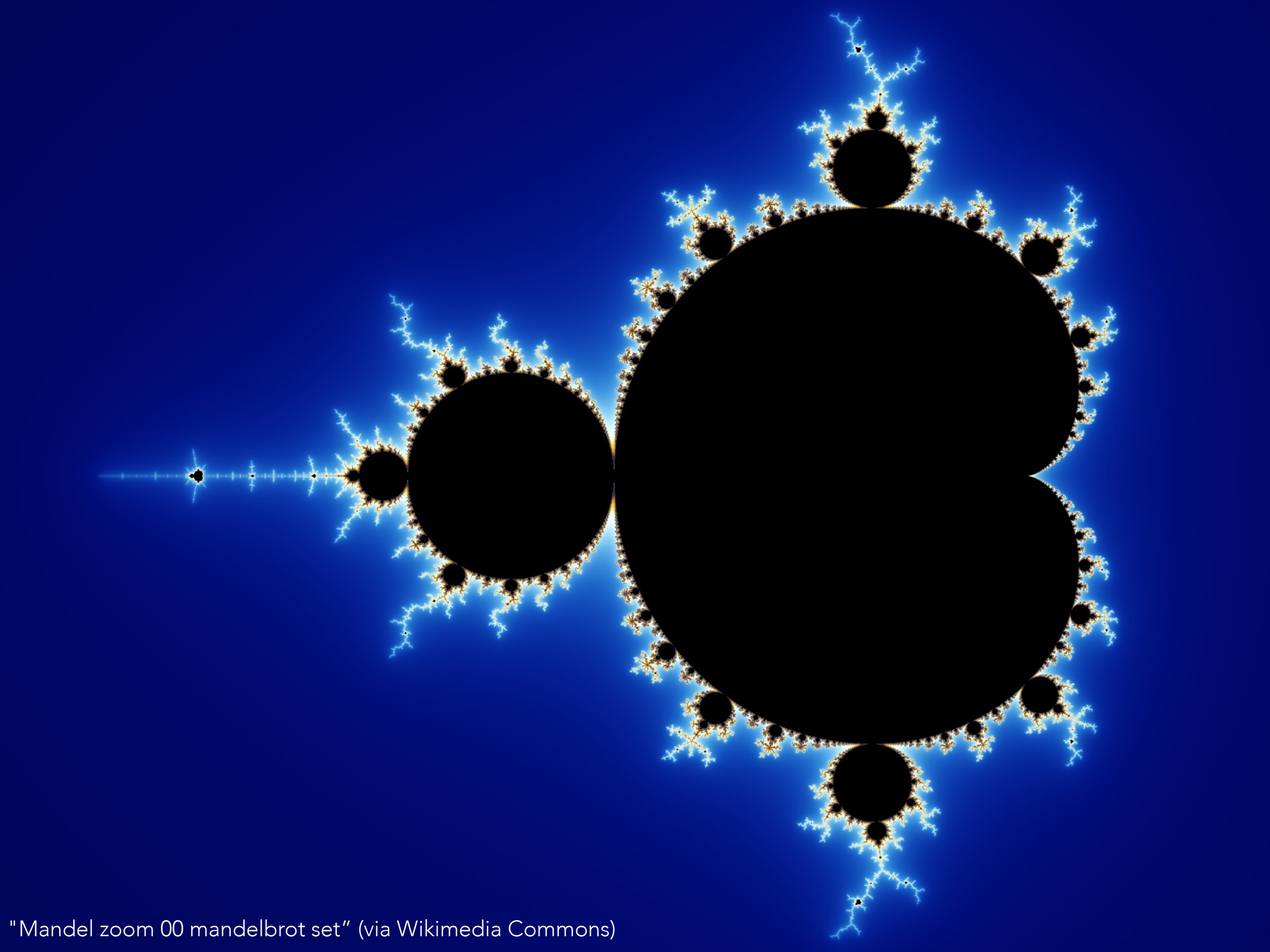
There is plenty of self-similarity…



Benoit Mandelbrot studied how simple processes could give rise to complex self-similar forms

Along the way, he coined the term “fractal” and established fractal geometry as a proper mathematical discipline.

He called it the “theory of roughness”, as opposed to classical – smooth – geometry



"Mandel zoom 00 mandelbrot set" (via Wikimedia Commons)

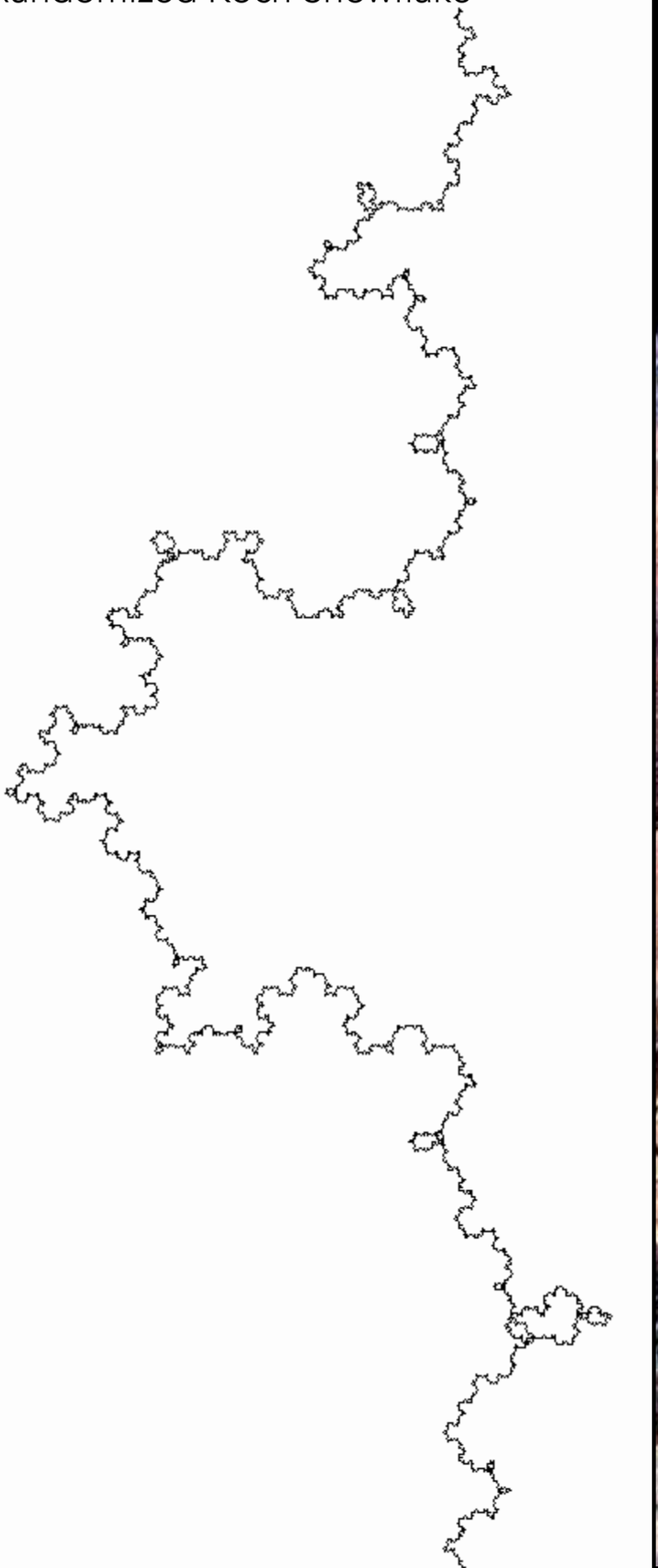
Himmalayas (Photo: NASA)

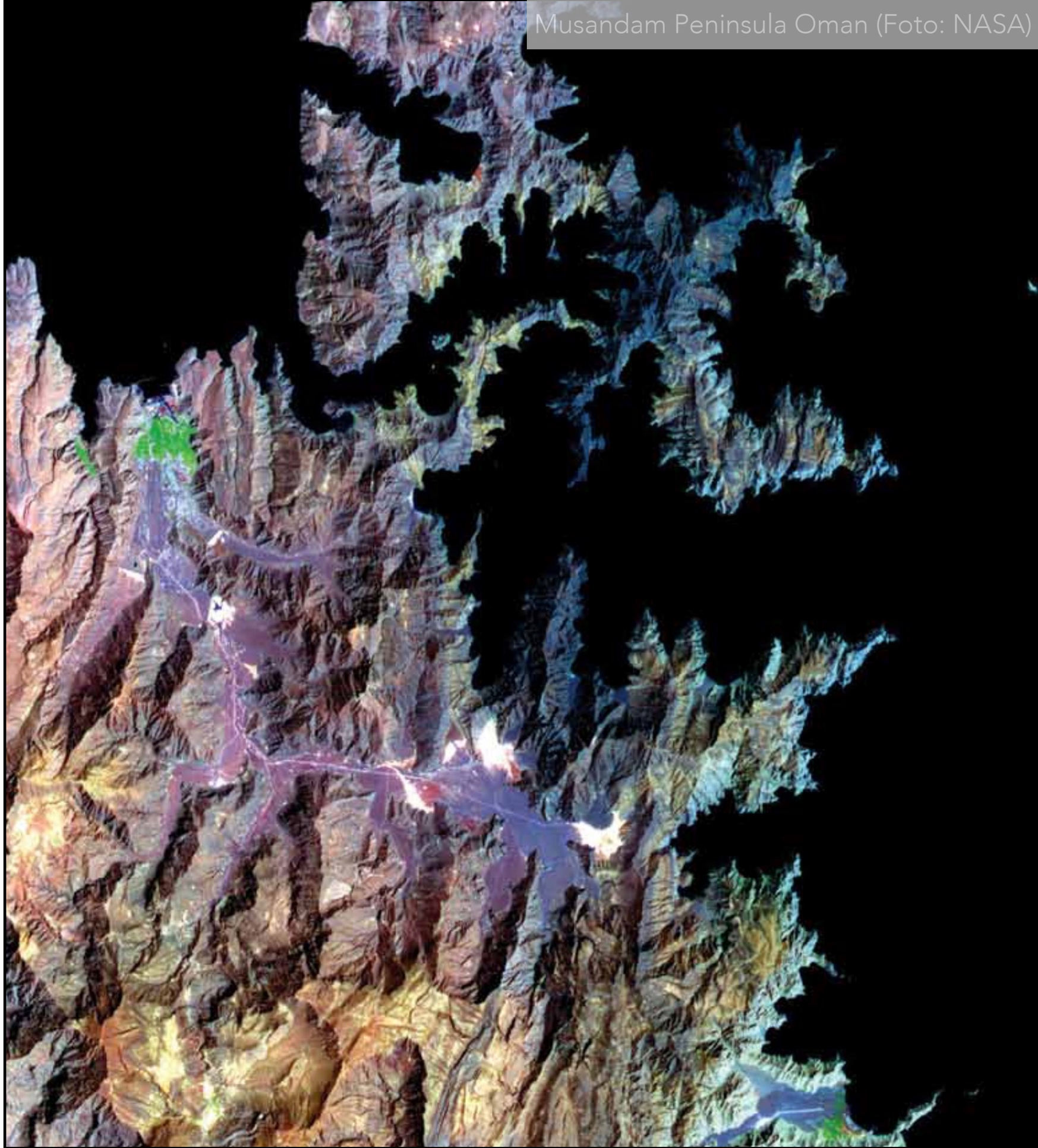
But how do the mathematical fractals connect to the "fractals" occurring in nature?

(They seem "too regular"…)

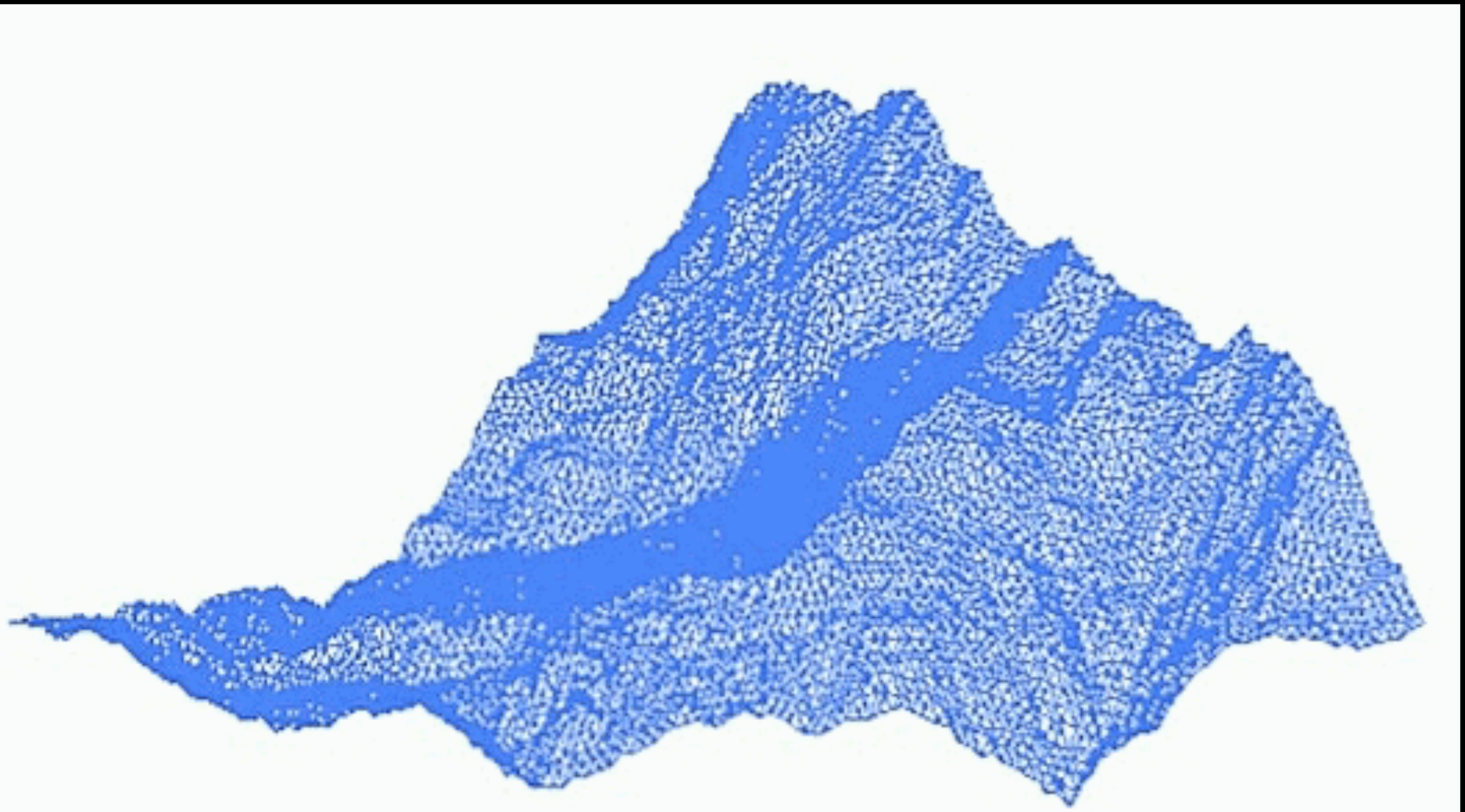
Enter: Randomness!

Randomized Koch Snowflake





Musandam Peninsula Oman (Foto: NASA)



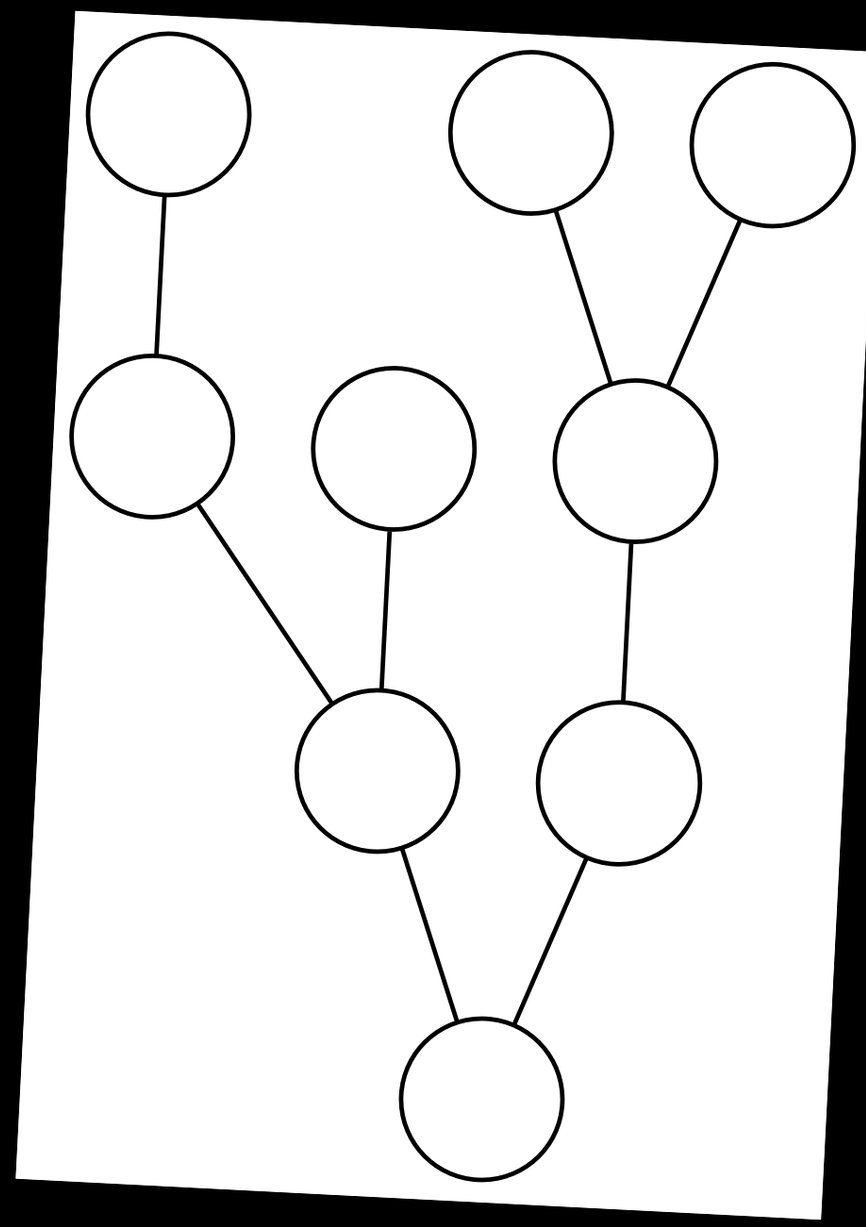
"We still share [Kepler's] belief in a mathematical harmony of the universe. It has withstood the test of ever widening experience. But we no longer seek this harmony in static forms like the regular solids, but in dynamic laws."

–HERMANN WEYL, *SYMMETRY*

Lena River Delta (Foto: NASA)

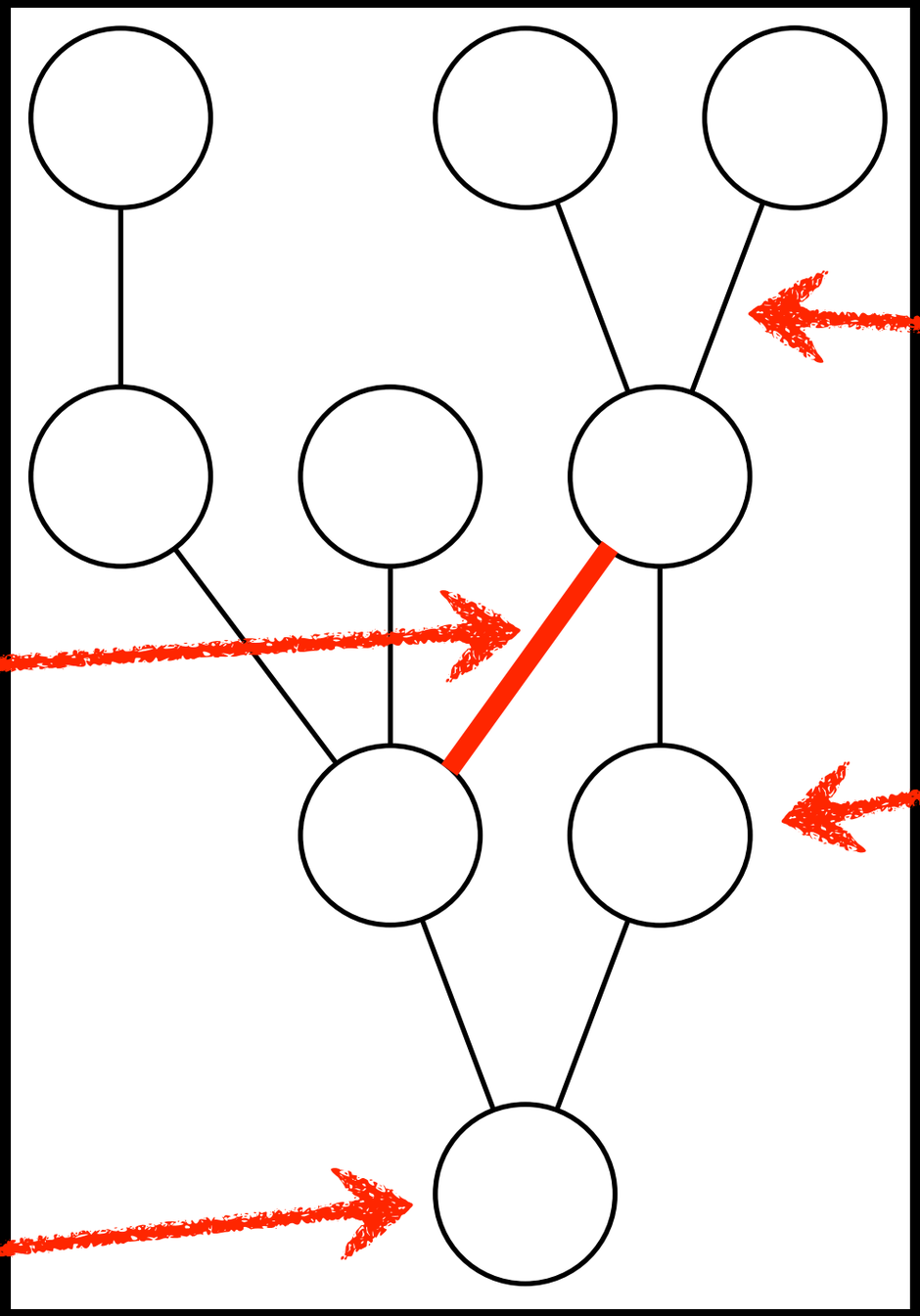
Digression: Trees

Real Tree



Mathematician's Tree

Mathematical Tree



Branching

Not allowed!

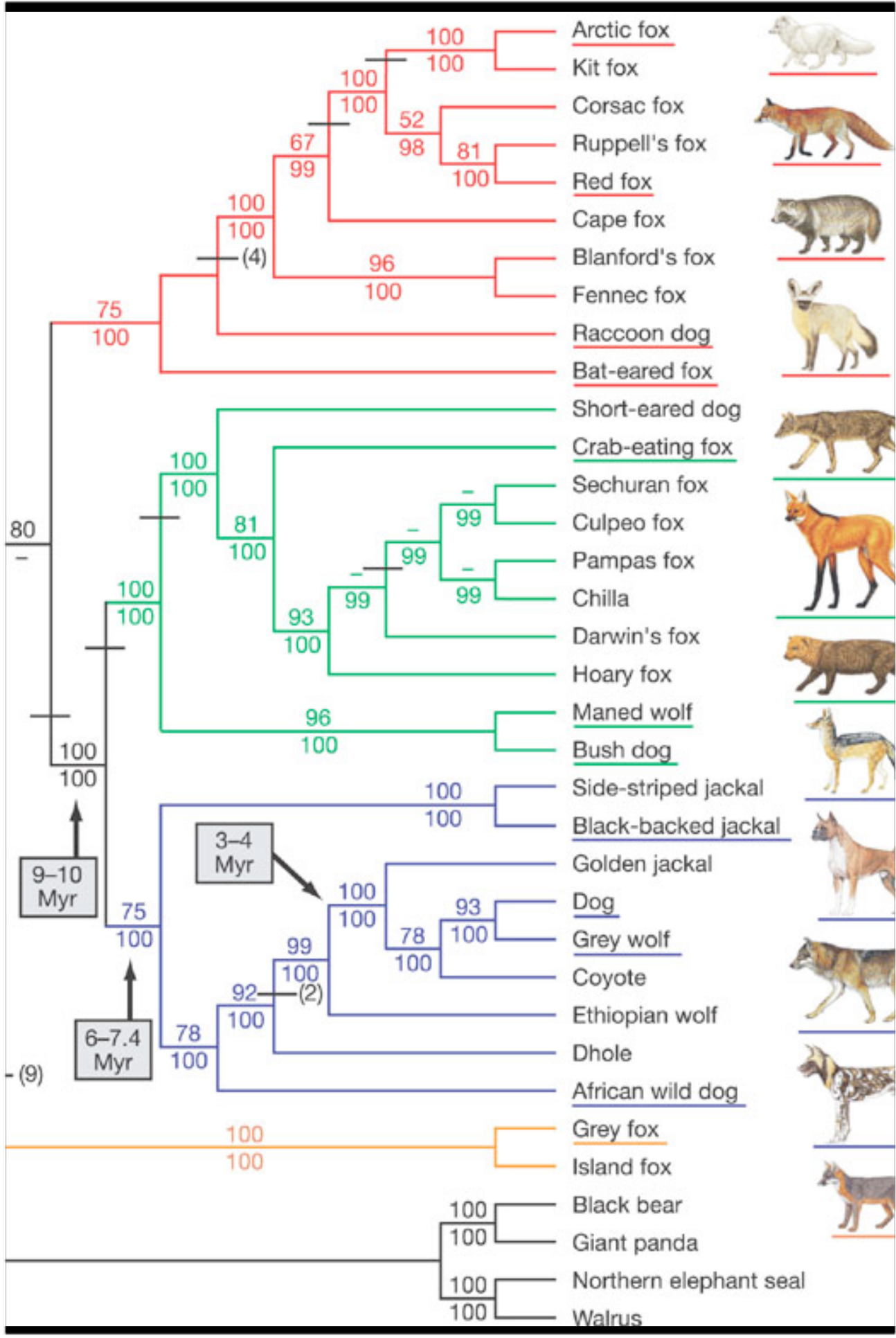
Node

Root

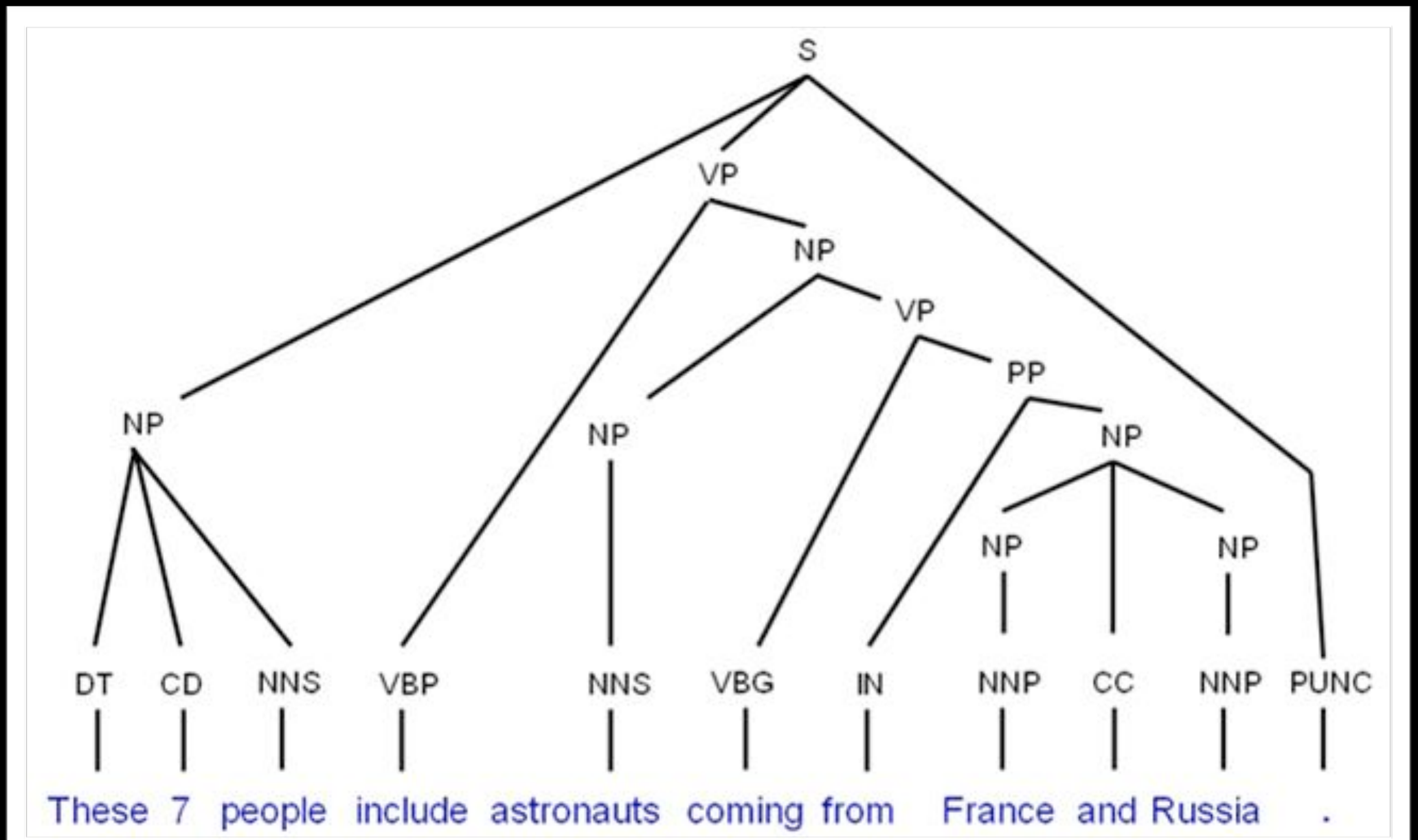
Trees are Plentyful

- In biology:
  phylogenetic trees are
  used to classify species (or
  other entities) according to
  common ancestors.

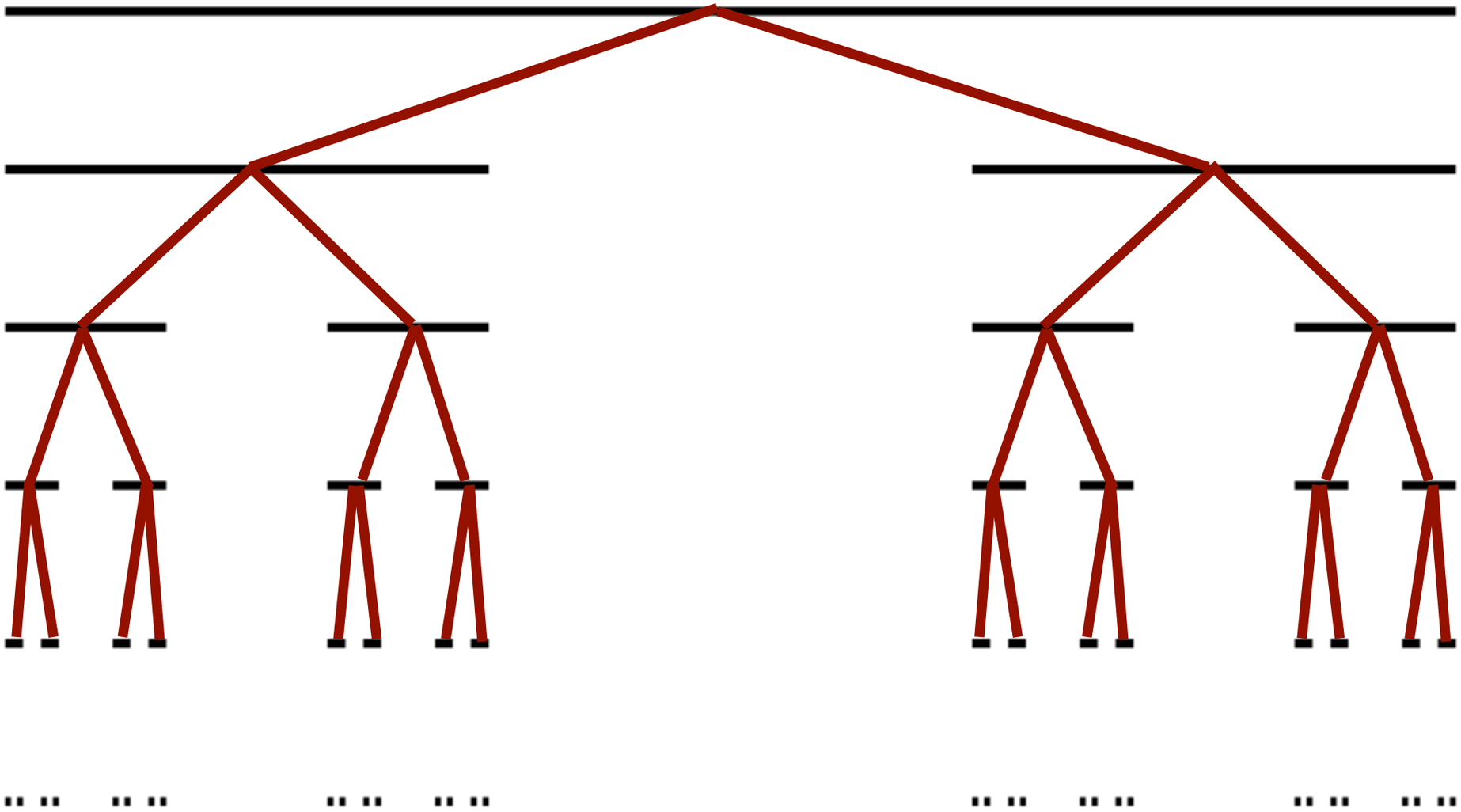
- Simple version:
  Family tree



Linguistics

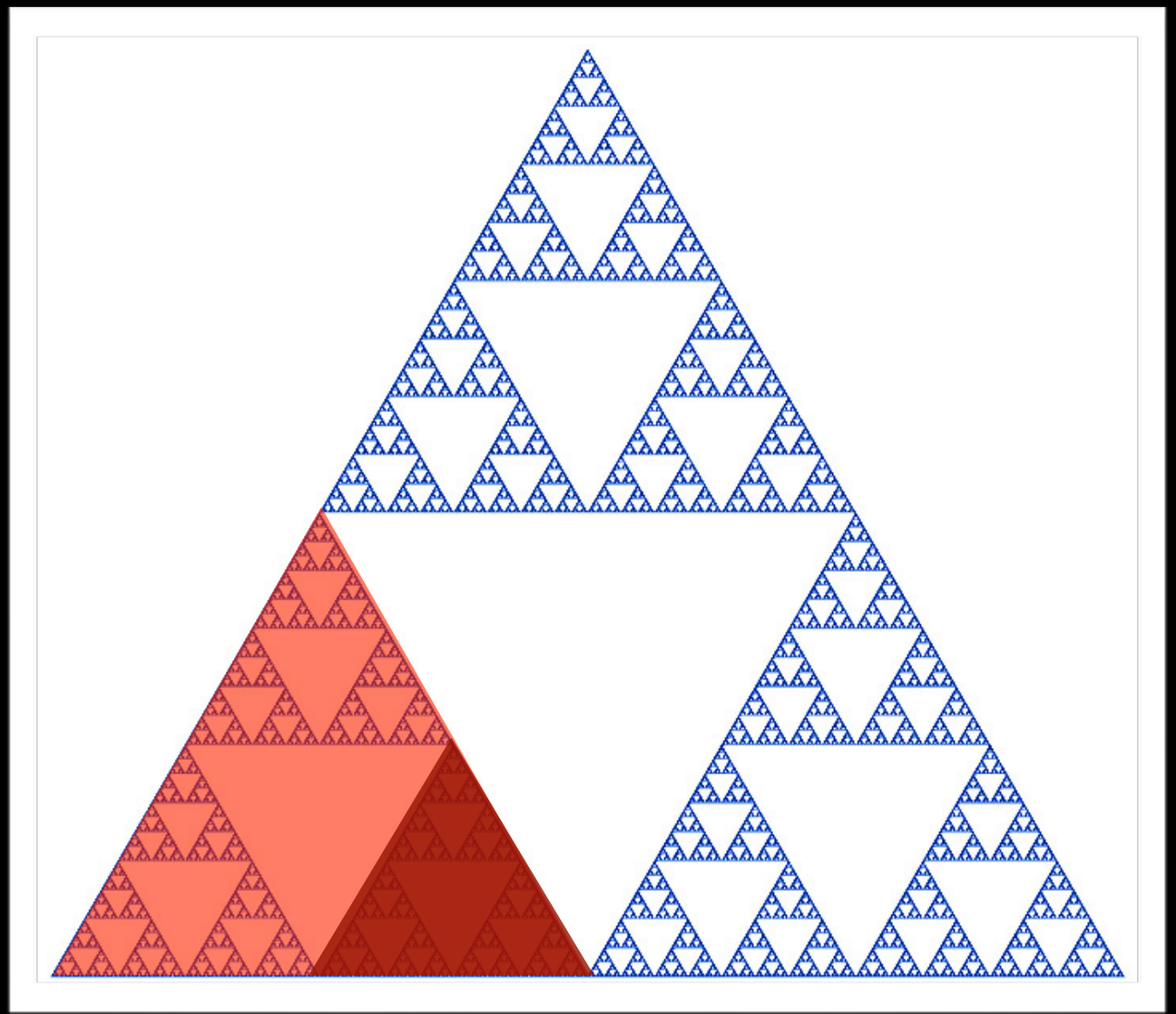


The Cantor Tree



The Sierpinski Triangle

1



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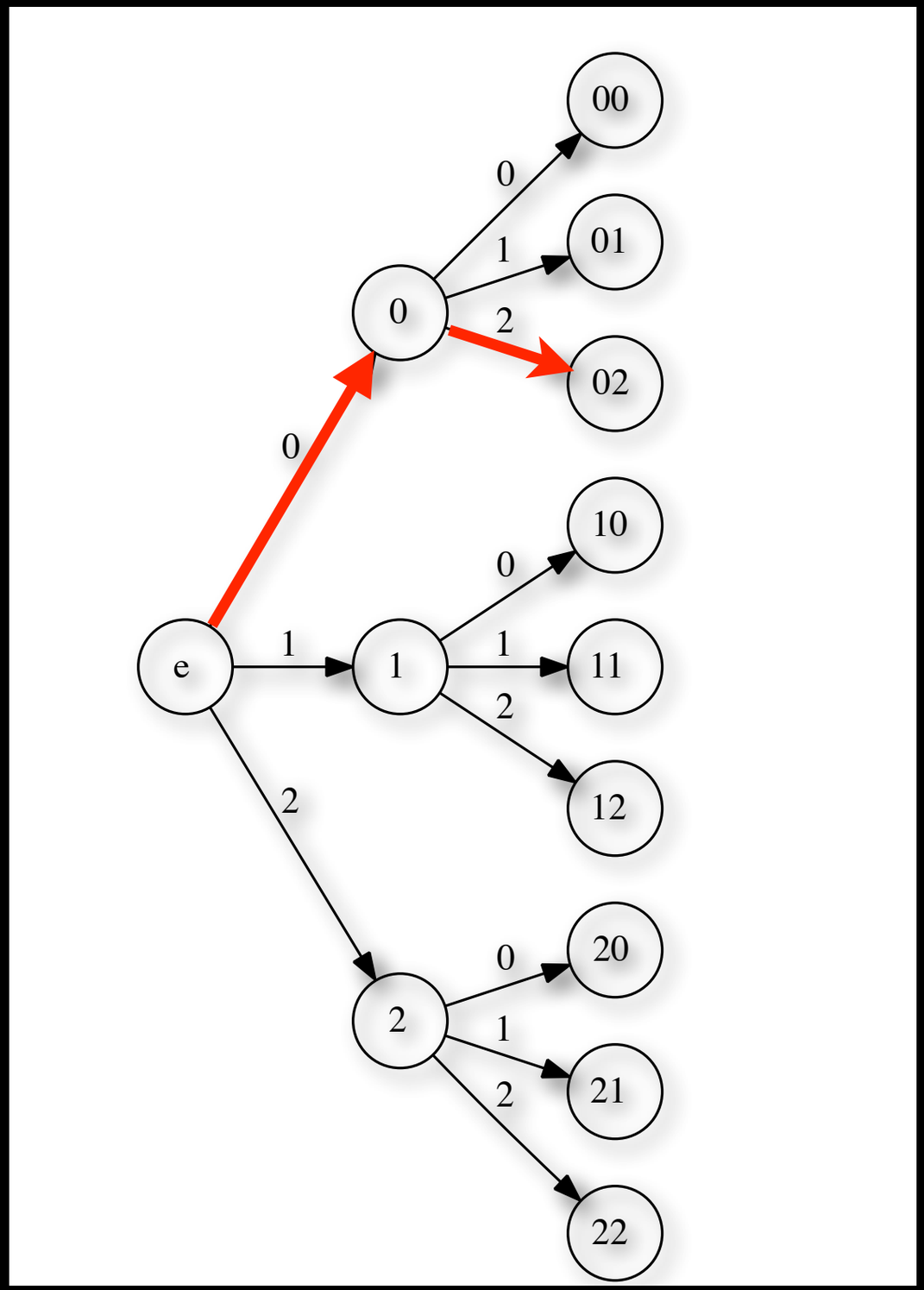
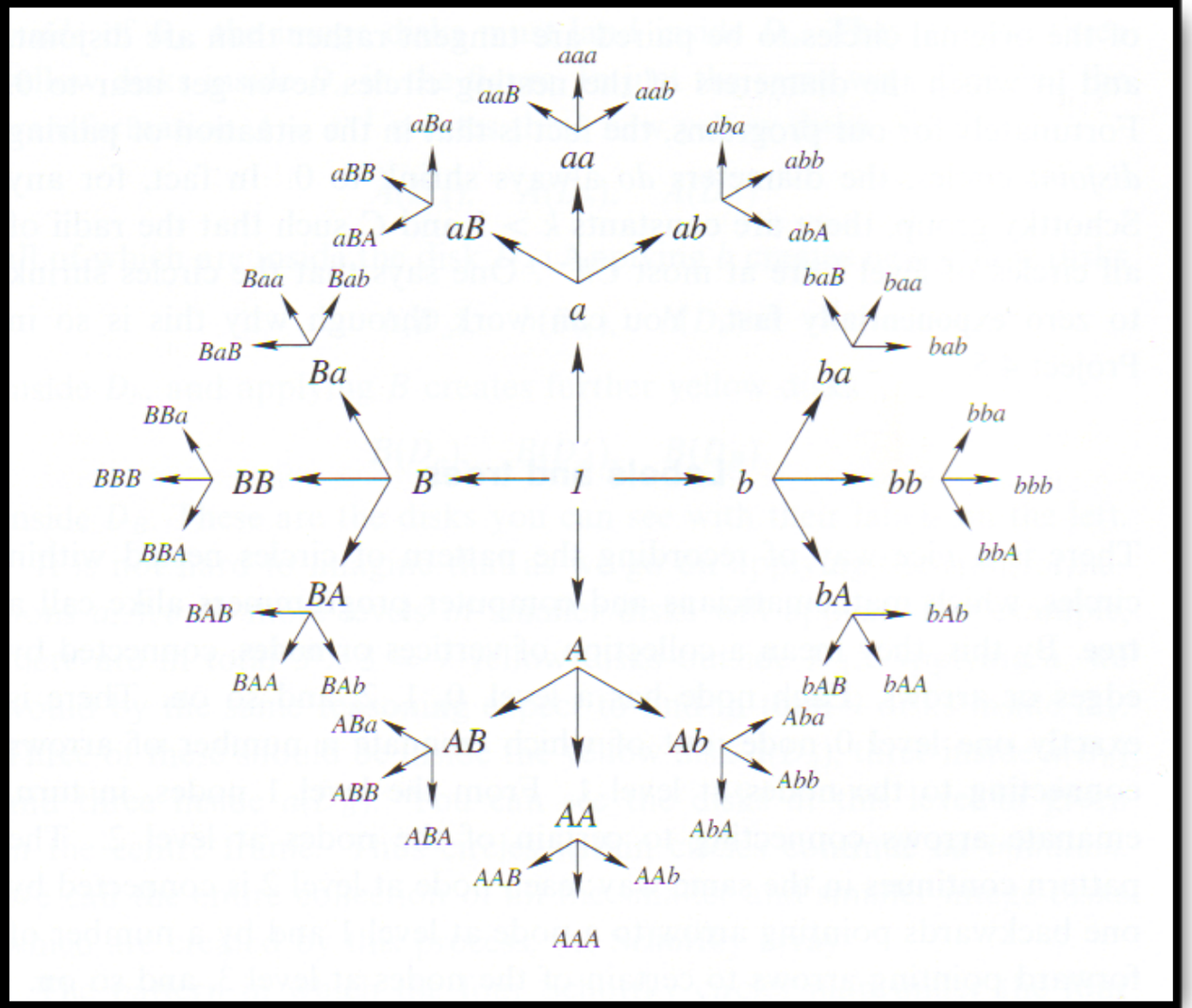
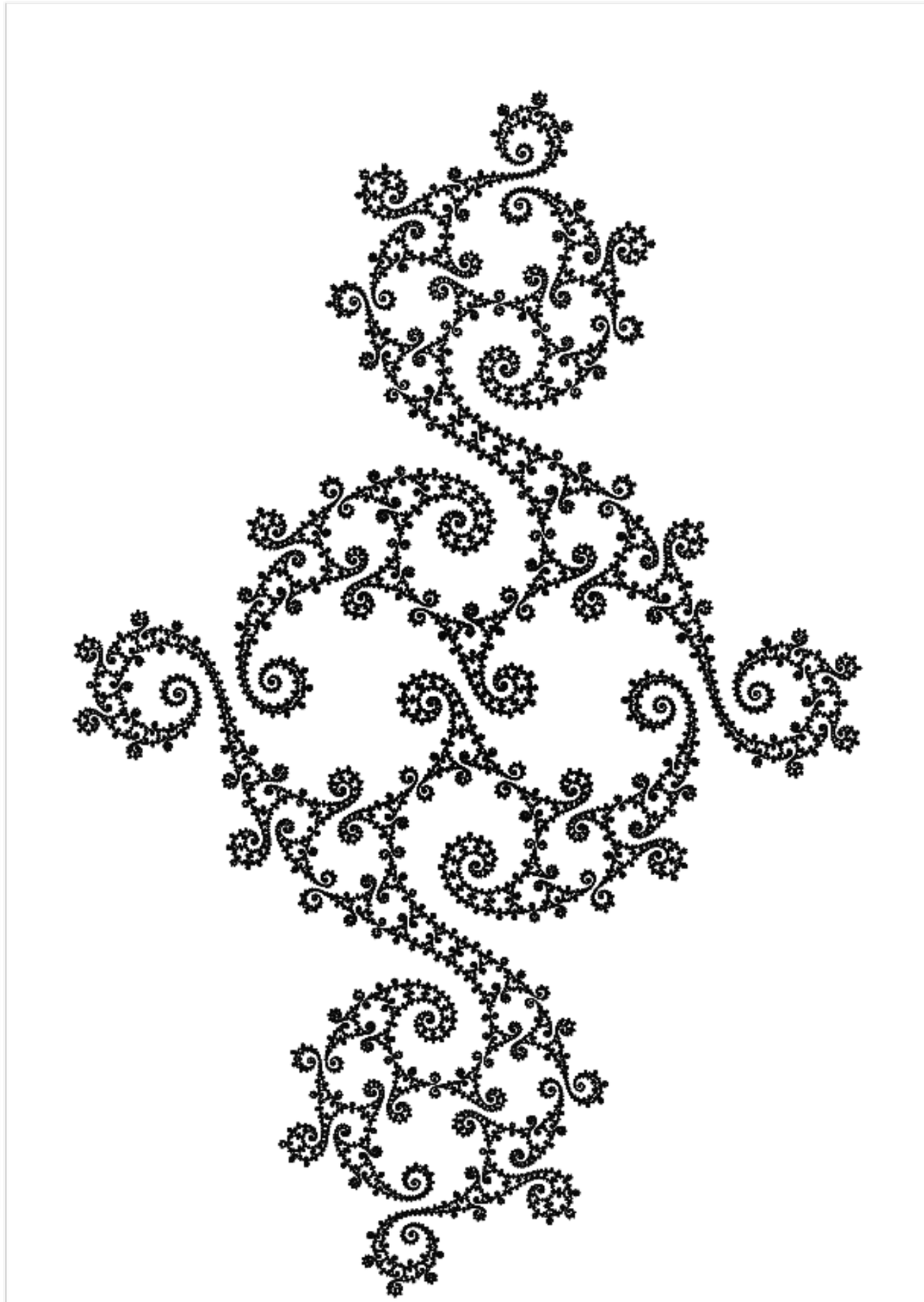


2

Coding Fractals





21ST CENTURY

=

THE INFORMATION AGE

Trees are a good example of a pattern that is interesting from a geometric as well as from a combinatorial point of view.

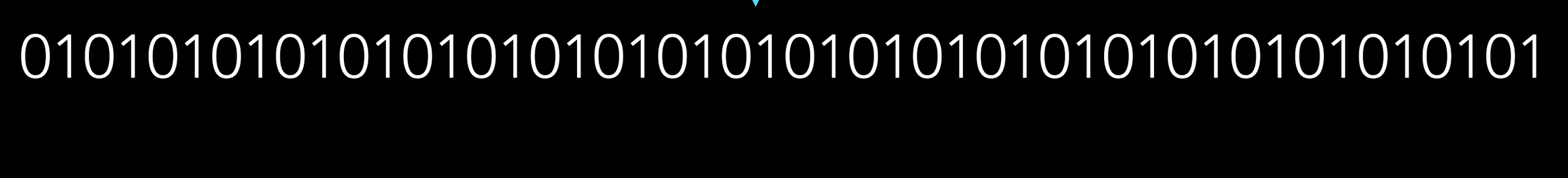
Today, in the age of big data, objects we are studying are often of this “mixed” nature

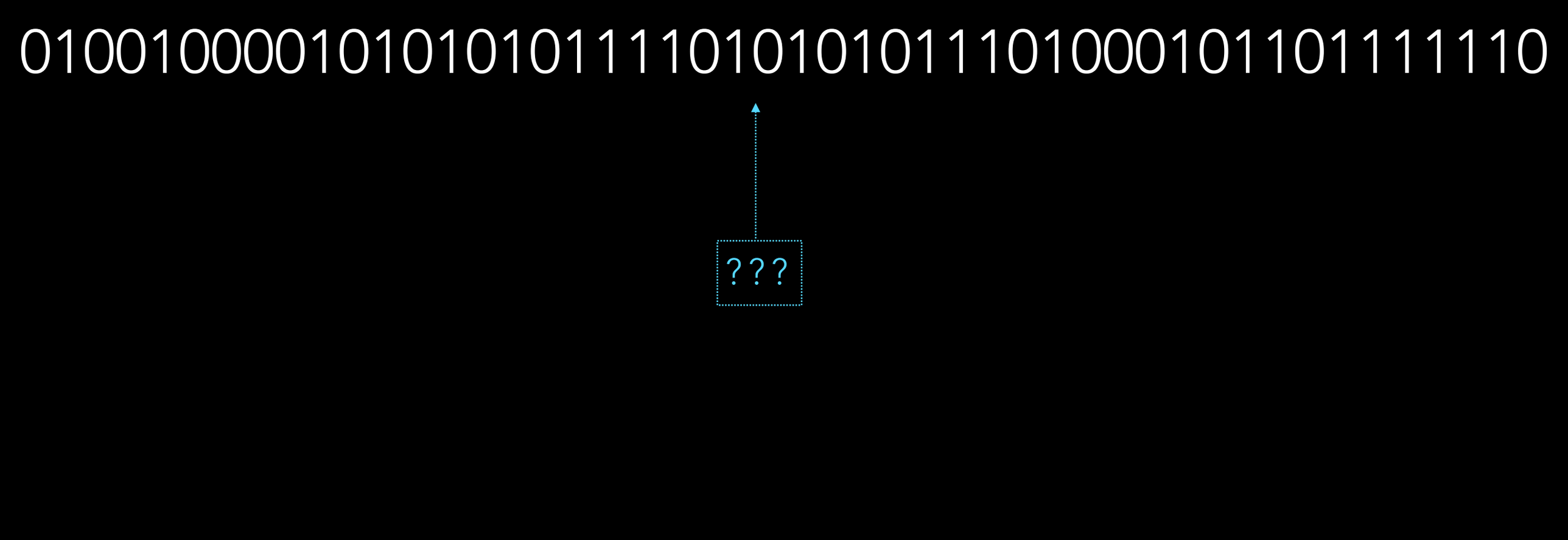
How do we find “patterns” here?

COULD WE LEAVE IT TO COMPUTERS?

Patterns = Data Compression

PRINT “01” 24 TIMES





Patterns = Data Compression

Compression ratio ~ 0.75

Compression ratio ~ 0.4

So, could we just start a computer program with the instruction:

FIND THE BEST POSSIBLE COMPRESSION (I.E. SHORTEST DESCRIPTION) OF OUR OBJECT.

The shortest description would then be
the ultimate pattern
of the object

We would have

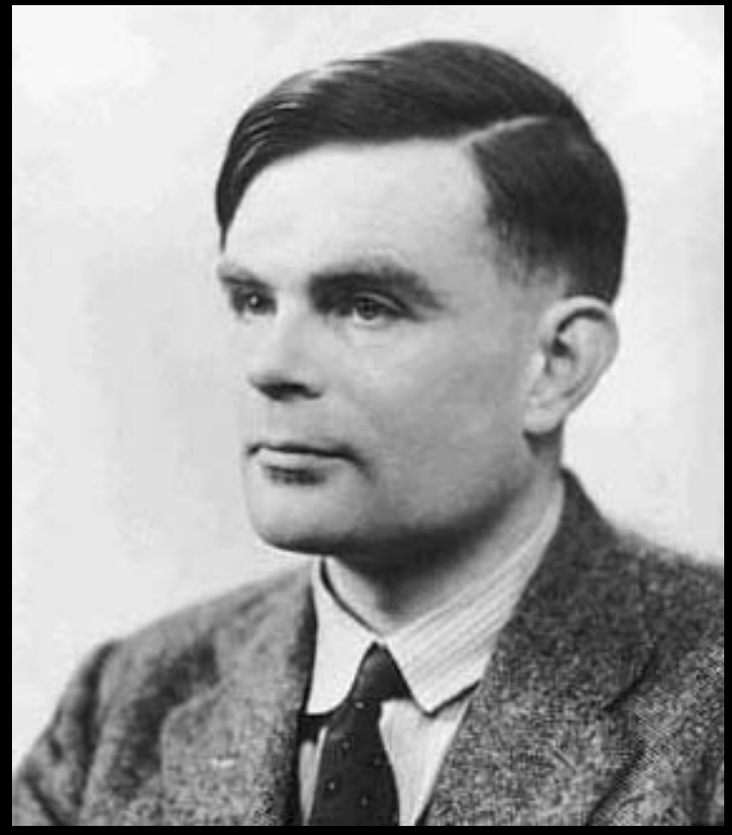
regular object = simple pattern = short descriptions

random noise = no pattern = no short description

"Unfortunately", such a computer program
cannot exist

The reason is that we can never be sure whether some compression is actually the shortest possible.

This is a consequence of the
Unsolvability of the Halting Problem,
shown by Alan Turing



“Mathematical thinking is, and must remain,
essentially creative.“

–EMILE POST, 1944

Thank you!

FOR SLIDES AND REFERENCES: JAN.REIMANN@PSU.EDU