

## The Model Existence Theorem

### Key Concepts

- Extend  $T$  to a Henkin theory  $T_H$ , then complete it to a theory  $T'$ .
- $T'$  is a complete Henkin theory for the extended language  $\mathcal{L}_H$ .
- For  $T'$ , its term model  $\mathcal{A}_{T'}$  satisfies

$$\mathcal{A}_{T'} \models \sigma \iff T' \vdash_{\mathcal{L}_H} \sigma$$

for all  $\mathcal{L}_H$ -sentences  $\sigma$ .

### Problems

#### Exercise 0.1.

How do we obtain a model for  $T$  from a model for  $T'$ ?

#### Exercise 0.2.

Why is  $\mathcal{A}_{T'}$  countable if  $\mathcal{L}$  is countable?

#### Exercise 0.3.

Let  $X$  be the set of all maximally consistent  $\mathcal{L}$ -theories. Recall that the sets

$$\langle \sigma \rangle = \{T \in X : \sigma \in T\} \quad (\sigma \text{ } \mathcal{L}\text{-sentence})$$

generate a Hausdorff topology on  $X$ .

Show that the topology is compact.